MSc EXAMINATION

ASTM-052 Extragalactic Astrophysics

Time Allowed: 1 hour 30 minutes

Date:

Time:

Answer TWO questions out of FOUR. Each question carries 25 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question. Calculators ARE permitted in this examination.

Physical and Astronomical Constants and Conversion Factors

Speed of light	c	3.00×10^{8}	m s^{-1}
Planck constant	h	6.63×10^{-34}	J s
Gravitational constant	G	6.67×10^{-11}	N m^{2} kg^{-2}
Boltzmann constant	k	1.38×10^{-23}	J K^{-1}
Stefan constant	σ	5.67×10^{-8}	W m^{-2} K^{-4}
Permeability of free space	μ_o	$4\pi \times 10^{-7}$	H m^{-1}
Thomson cross section	σ_T	6.7×10^{-29}	m^{2}
Proton rest mass	m_p	1.673×10^{-27}	kg
Mass of sun	M_{sun}	1.99×10^{30}	kg
Radius of sun	R_{sun}	6.96×10^{8}	m
Luminosity of sun	L_{sun}	3.9×10^{26}	W
Distance of earth from sun	1 AU	1.50×10^{11}	m
Hubble constant	H_o	$100 \times h$	km s^{-1} Mpc^{-1}
Year	1 y	3.16×10^{7}	s
Electron volt	1 eV	1.6×10^{-19}	J
Jansky	1 Jy	10^{-26}	W m^{-2}
Parsec	1 pc	3.085×10^{16}	m

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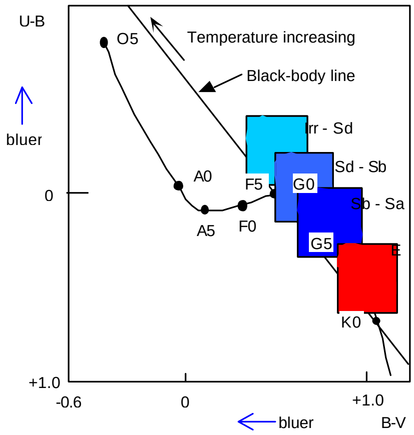
1. Very briefly describe Baade's classification of stars into Populations I and II. **[3 marks]**



The figure shows the regions of the colour-colour diagram occupied by the different Hubble types of galaxies, together with the locus of main-sequence stars. Discuss briefly what this tells us about the stellar populations of the various galaxy types. **[5 marks]**

Briefly describe how the masses of (a) elliptical and (b) spiral galaxies can be determined.

[5 marks]

In the solar neighbourhood, the stellar mass-function $f(M)$ is given by

$$f(M) = f_*\left(\frac{M}{M_*}\right)^{-a}; \quad a \sim 2.5,$$

where M is the mass of a star and f_* and M_* are constants. Show that the total mass density M_{total} of stars per unit volume is given by

$$M_{\text{total}} \approx \frac{f_* M_*^2}{(a-2)}\left(\frac{M_{\text{low}}}{M_*}\right)^{2-a},$$

where M_{low} is the lower cut-off of the mass spectrum. **[6 marks]**

It can be shown that the total luminosity-density L_{total} of stars in the solar neighbourhood is given by

$$L_{\text{total}} \approx \frac{f_* M_* L_*}{(4.3-a)}\left(\frac{M_{\text{high}}}{M_*}\right)^{4.3-a},$$

where M_{high} is the upper cut-off of the mass spectrum. Discuss the difficulty of using the last two results to obtain the mass-luminosity ratio of stars in spiral galaxies. **[3 marks]**

An estimate of the average mass-luminosity ratio of stars in spiral galaxies is 2, in solar units. Comment on this value in comparison with the observed mass-luminosty ratios of such galaxies. **[3 marks]**

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2. State the Jeans condition for collapse of an interstellar cloud of gas in terms of its free-fall time t_{ff} and the time t_s taken for sound to cross the cloud. **[3 marks]**

   The rotation curve $\Theta(r)$ of a spiral galaxy is well approximated by

$$\Theta(r) = \begin{cases} \dfrac{r}{r_o} \times \Theta_o & r < r_o \,; \\ \Theta_o & r \geq r_o \,, \end{cases}$$

   where r is the distance from the centre of the galaxy and Θ_o is a constant. Show that the density of the halo must follow the law

$$r(r) \begin{cases} = \text{constant} & r < r_o \,; \\ \propto \dfrac{1}{r^2} & r >> r_o \,. \end{cases}$$

   [7 marks]

   The epicyclic angular frequency $k(r)$ is given by

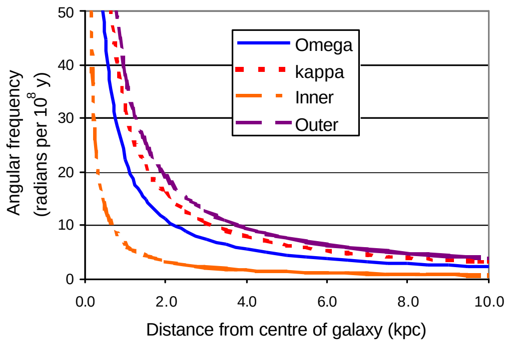
$$k^2(r) = -4B(r)[A(r) - B(r)]$$

   where the Oort parameters $A(r)$ and $B(r)$ are given by

$$A(r) = +\frac{1}{2}\left[\frac{\Theta(r)}{r} - \frac{d\,\Theta(r)}{dr}\right]; \quad B(r) = -\frac{1}{2}\left[\frac{\Theta(r)}{r} + \frac{d\,\Theta(r)}{dr}\right].$$

   Show that the epicyclic orbits in the outer regions of the above galaxy are not closed in an inertial frame. **[7 marks]**

   Why do the *Lindblad resonances* set limits on the extent of spiral structure? **[4 marks]**



   The figure shows the angular and epicyclic frequencies *Omega* and *kappa* of a disc galaxy as a function of distance from its centre, together with the inner and outer Lindblad resonances. Over approximately what range of distance can a density-wave of angular frequency 20 radians per 10^8 years exist in the galaxy? **[4 marks]**

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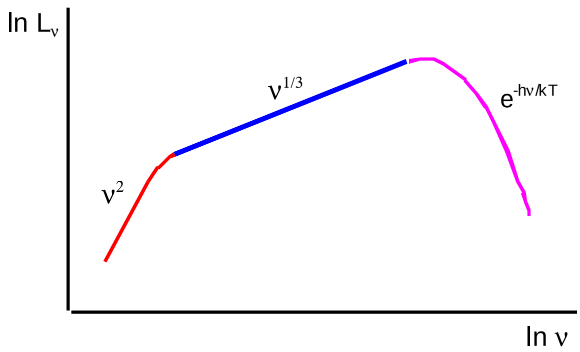
3. Explain briefly and qualitatively why there is a limit to the luminosity that can be generated by accretion. **[5 marks]**

The luminosity $l(r)$ per unit area of an accretion disc around a black hole of mass M, at distance r from the disc's centre, is given by

$$l(r) = \frac{3}{8\pi}\frac{GM\dot{m}}{r^3}\left[1-\left(\frac{r_{\rm lso}}{r}\right)^{1/2}\right],$$

where $\dot{m}$ is the rate of accretion of mass through the disc. What is the significance of $r_{\rm lso}$ and why does the luminosity vanish there? **[5 marks]**

Assuming that the accretion disc radiates as a black body, estimate the maximum temperature occurring in a disc, around a black hole of mass 10^8 $M_{\rm sun}$, accreting material at the rate of one solar mass a year, and calculate the typical energy of the corresponding emitted photons. **[12 marks]**



A very simple model of the spectrum emitted by such an accretion disc is shown in the figure. Explain very briefly, and without derivation, the origin of the three parts of this spectrum. **[3 marks]**

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4. Describe briefly the various types of active galaxy. **[5 marks]**

The power $P(E)$ radiated by an electron of energy E in a magnetic field B is given by

$$P(E) = bB^2E^2 \,,$$

where b is composed of fundamental constants. This power is predominately emitted at frequency n given by

$$n = aBE^2$$

where a is also composed of fundamental constants. Show that, if there are $N(E)dE$ such electrons with energies in the range E to $E + dE$, where

$$N(E) = N_o \left(\frac{E}{E_o} \right)^{-p}$$

the luminosity $L(n)$ of the assembly at frequency n is given by

$$L(n) = \frac{1}{2} N_o E_o^p \left[\frac{b}{a^{(3-p)/2}} \right] B^{\left(\frac{1+p}{2}\right)} n^{\left(\frac{1-p}{2}\right)} .$$ **[7 marks]**

For many sources, typical values of spectral index are around minus 0.7. Comment on the equivalent value of p. **[3 marks]**

Show that the characteristic lifetime $t(E)$ of an electron of energy E is given by

$$t(E) = \frac{1}{bB^2E} = \left(\frac{a^{1/2}}{b} \right) B^{-3/2} n^{-1/2} .$$ **[5 marks]**

where n is the frequency at which the electron radiates most of its energy.

Numerically, the above equation for the lifetime can be written as

$$t(\mathrm{s}) = 6 \times 10^5 \, B(\mathrm{T})^{-3/2} n(\mathrm{Hz})^{-1/2} .$$

The lobes of a radio source are mapped at 6-cm wavelength and are found to lie 50 kpc from the central galaxy. They are deduced to contain a field of 10^{-8} tesla. Comment on the supply of electrons to the lobes. **[5 marks]**

End of Examination Paper **Peter Clegg**