Queen Mary, University of London

ASTM 002 Galaxies

Wednesday June 4, 2003, 6.15 pm

Time Allowed: 3 hours

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination.

continued overleaf ...

1. Explain, in one or two sentences, what *crossing time* and *relaxation time* mean for a system of stars. [3 marks]

Two stellar-dynamical systems, a globular cluster and a galaxy, are to be simulated over 10^{10} yr using the same number of particles. These have

	size	stars	typical velocity
globular cluster	50 pc	10^6	50 km s^{-1}
galaxy	20 kpc	10^{10}	200 km s^{-1}

Explain which system is more difficult to simulate, and why. [10]

A spherical galaxy has a total density distribution

$$\rho_{\rm tot}(r) = \rho_0 a^2 r^{-2}$$

A particular stellar population in the galaxy has a distribution that is spherical, non-rotating, isothermal and isotropic, with velocity dispersion σ in each velocity component; its mass is negligible. Derive the radial density distribution of this population and show that it depends on a dimensionless number. Give a physical interpretation of this number. [12]

[10^{10} yr $\simeq 3 \times 10^{17}$ s, 1 pc $\simeq 3 \times 10^{13}$ km. The Jeans equation for a spherical system is

$$\frac{d}{dr}\left(\rho\sigma_{rr}\right) + \frac{\rho}{r}\left[2\sigma_{rr} - \left(\sigma_{\theta\theta} + \sigma_{\phi\phi} + \langle v_\phi \rangle^2\right)\right] = -\rho\frac{d\Phi}{dr}$$

in the usual notation.]

2. What is the Orion nebula, and what is the origin of the red and green colours in it? [6]

Explain the physical process through which the red emission is related to the UV continuum from the young stars. [10]

Consider a system such as the Orion nebula, having total mass $M_{\rm total}$, of which $M_{\rm stars}$ is in stars and $M_{\rm gas}$ is in gas, and of the latter $M_{\rm metal}$ is in metals. $Z \equiv M_{\rm metal}/M_{\rm gas}$ is the metallicity. Let the differentials $\delta M_{\rm stars}$ and so on denote changes due to one generation of stars.

Express δZ in terms of $M_{\rm metal}$, $M_{\rm gas}$ and their differentials. [4]

Assuming constant yield p, and that no material enters or leaves the region, show that

$$Z = -p \ln\left(\text{gas fraction}\right) \quad [5]$$

3. If the Local Group (mass $\sim 10^{12} M_\odot$ and size ~ 1 Mpc) is viewed from a cosmological distance of 10^9 pc, estimate (a) the angular size and (b) the Einstein radius for sources at infinity. Does the system make an effective gravitational lens? [6]

Some theories for dark matter in galaxies involve gas clouds of mass $\sim 10^{-3} M_\odot$ and size ~ 30 AU. At what distances would such gas clouds become effective lenses? [7]

If our Galaxy has an isothermal halo (with velocity dispersion σ) which consists of machos, show that for microlensing programs observing through the halo, the covering factor of Einstein rings will be of order σ^2/c^2. [8]

In a survey of objects with luminosity function $f(L)$ to a limit of $L_{\min}$, the number of objects detected will be

$$\int_{L_{\min}}^{\infty} f(L)\, dL \times \langle \text{area of survey} \rangle \,.$$

Now suppose there is a foreground lens in the survey area with uniform scalar magnification $|M|$. Explain how the above formula must be modified. [4]

[The Einstein radius for $1M_{\odot}$ at 1 pc with a source at infinity is $\simeq 0.1$ arcsec or $\simeq 0.5 \times 10^{-6}$ rad.

An isothermal sphere has density $\rho = \sigma^2/(2\pi G r^2)$, where σ is the effective dispersion.

The Einstein radius of a point mass M is $\left[(4GM/c^2)(D_{\rm L}D_{\rm LS}/D_{\rm S})\right]^{1/2}$ where $D_{\rm L}$ is the distance to the lens, $D_{\rm S}$ to the source, and $D_{\rm LS}$ the distance from the lens to the source.]

4. The distance l from the Milky Way to Andromeda or a distant Local Group dwarf galaxy satisfies the differential equation

$$\frac{d^2 l}{dt^2} = -\frac{GM}{l^2}$$

with M being total mass Local Group.

Verify that

$$t = \tau_0(\eta - \sin\eta),$$
$$l = \left(GM\tau_0^2\right)^{\frac{1}{3}} (1 - \cos\eta)$$

and

$$t = \tau_0(\sinh\eta - \eta),$$
$$l = \left(GM\tau_0^2\right)^{\frac{1}{3}} (\cosh\eta - 1)$$

are both solutions of the differential equation (τ_0 being an integration constant). [8]

Explain the physical difference between these two solutions. Why does each solution have only the one integration constant τ? [4]

There exists still another solution, which has the form

$$l = At^n.$$

Find this solution and explain its physical significance. [13]

End of Examination

P. Saha
C.D. Murray