Queen Mary, University of London

ASTM 002 Galaxies

Friday May 24, 2002

Time Allowed: 3 hours

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination.

continued overleaf ...

1. Briefly explain the terms *crossing time* and *relaxation time* in stellar systems. [4 marks]

The relaxation time may be expressed as

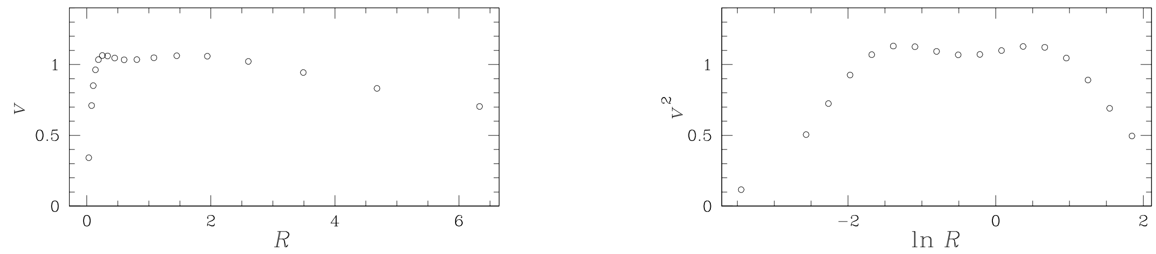
$$\frac{1}{8N \ln N} \frac{(Rv)^3}{(Gm)^2} \qquad \text{or} \qquad \frac{N}{8 \ln N} \frac{R}{v},$$

R being the size of the system, N the number of stars, m their typical mass, v their typical velocity and G the gravitational constant.

Derive the v and m dependences of the first form of this formula using a back of the envelope calculation. [8]

Suppose some intelligent dinosaurs had succeeded in measuring distances and velocities of stars, and left us a record—from some epoch when the Sun was closer to the Galactic centre than now—of stellar distances and velocities in the solar neighbourhood. What would be expect for the old real- and velocity-space densities as compared to the current values? If the numbers contradicted the expectation, what would we conclude? [5]

The two panels below plot (in two different ways) the rotation curve $v(R)$ of a spiral galaxy. The first hump in the rotation curve comes from a disc with scale radius R_0, the second hump from a dark halo. Make a rough estimate of R_0 (or alternatively of $\ln R_0$) in the units of the figure. Justify your answer. (Detailed calculation not required—an explanation and/or sketch is sufficient.) [8]



2. Consider a star-forming region with stellar mass M_{stars} and gas mass M_{gas}; of the latter M_{metal} is in metals. Let the differentials δM_{stars} and so on denote changes due to one generation of stars.

In this context, what does *yield* express? [2]

Denoting metallicity by $Z \equiv M_{\text{metal}}/M_{\text{gas}}$, express δZ in terms of M_{metal}, M_{gas} and their differentials. [4]

Assuming constant yield p, and that no metal enters or leaves the region, relate δM_{metal} and δM_{stars}. Allowing for (metal-free) gas to be accreted into the system, relate δM_{stars} to δM_{gas} and the total mass. [6]

Use the above to show that

$$\delta Z = \frac{(p - Z)\delta M_{\text{total}} - p\delta M_{\text{gas}}}{M_{\text{gas}}}. \qquad [2]$$

In the simple case where the gas accretion rate equals the rate of mass being locked up in stars and stellar remnants, show that Z asymptotes to p. [6]

Explain physically why we should expect such behaviour for stellar metallicities in this case. [5]

3. Gravitational lensing in the Milky Way halo is considered a test of whether the halo mass is made of wimps or machos. But lensing only depends on mass, so it would seem not to care whether the mass is in wimps or machos. Resolve this apparent contradiction. (No calculation required.) [3]

What does microlensing optical depth τ mean? Derive the formula

$$\tau = \frac{4\pi G}{c^2 D_\mathrm{S}} \int_0^{D_\mathrm{S}} D_\mathrm{L} D_\mathrm{LS}\, \rho(D_\mathrm{L})\, dD_\mathrm{L}$$

for the microlensing optical depth. [10]

Imagine an observer at radius $r = 1$ in an isothermal sphere made of machos, looking outwards (i.e., towards the anti-centre) at sources at radius $r = a$, and monitoring for microlensing. Show that τ for this observer will be

$$\tau = 2\frac{\sigma^2}{c^2}\left[\frac{a+1}{a-1}\ln a - 2\right] \qquad [6]$$

Now consider observers on Earth monitoring stars 60 kpc away towards the Galactic anti-centre for microlensing. How many stars (order of magnitude) should a reasonable survey program monitor? [6]

[An isothermal sphere has density $\rho = \sigma^2/(2\pi G r^2)$, where σ is the effective dispersion. The Einstein radius of a point mass M is $\left[(4GM/c^2)(D_\mathrm{L} D_\mathrm{LS}/D_\mathrm{S})\right]^{1/2}$ where D_L is the distance to the lens, D_S to the source, and D_LS the distance from the lens to the source. $\int_1^a x^{-2}(x-1)(a-x)\,dx = 2(1-a) + \ln a(1+a)$.]

4. The distance l between the Milky Way and Andromeda satisfies the differential equation

$$\frac{d^2 l}{dt^2} = -\frac{GM}{l^2},$$

M being the total mass.

Verify that

$$t = \tau_0(\sinh\eta - \eta),$$
$$l = \left(GM\tau_0^2\right)^{\frac{1}{3}}(\cosh\eta - 1)$$

is a solution of the above equation (τ_0 being a constant of integration). [5]

Why is there only one integration constant. [3]

However this solution cannot be correct for the Milky-Way-Andromeda system. It makes an observational prediction that is completely wrong. What observation does it mis-predict, and what is the physical significance? [5]

Suppose the particular observation in question had been different, and in agreement with the above solution. Explain (without numerical calculation) how one would compute M. [12]

End of Examination

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