Queen Mary
University of London

BSc/MSci Examination

PHY214 Thermal and Kinetic Physics

Time allowed: 2hours 30 minutes

Date: ???? 2011

Time: ??:?? – ??:??

Instructions:

Answer ALL questions in section A. Answer ONLY TWO questions from section B. Section A carries 50 marks, each question in section B carries 25 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question. Course work comprises 20% of the final mark

Data. A sheet with useful physical values that may be of help in various questions is provided at the end of the examination paper.

Numeric calculators are permitted in this examination. Please state on your answer book the name and type of machine used. Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Important note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from the college. Please check now to ensure you do not have any notes in your possession. If you have any then please raise your hand and give them to an invigilator immediately. Exam papers cannot be removed from the exam room

You are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Examiners: K.J.Donovan, T.J.S.Dennis

SECTION A. Answers all questions in Section A

Question A1

Write down the first law of thermodynamics in infinitesimal form for a P-V system ***using only state variables***. Explain all of the symbols used. **[5]**

Question A2

Write down the 2nd Law of Thermodynamics expressed as an inequality involving only state variables. **[5]**

Question A3

A Carnot refrigerator operates between hot and cold reservoirs at temperatures T_1 and T_2 respectively. Write down the efficiency of the refrigerator in terms of these temperatures. **[5]**

Question A4

Write down Boltzmann's equation describing the relationship between entropy, S, and the number of distinct microstates, Ω, that can make the macrostate in question. **[5]**

Question A5

A heat pump operating between a hot and cold reservoir accepts heat Q_2 from the cold reservoir and givens up heat Q_1 to the hot reservoir. Write down the heat pump efficiency, η_{HP}, in terms of these heat flows. **[5]**

Question A6

During an adiabatic process on an ideal gas what is the relationship between initial temperature and volume T_i and V_i and final temperature and volume T_f and V_f? Explain any terms used. **[5]**

Question A7

Express the heat capacity of a gas at constant pressure, C_P, in terms of a partial differential ***using only state variables***. Define all of the symbols used. **[5]**

Question A8

What is the enthalpy, H, of a gas of N vibrating diatomic molecules at equilibrium at temperature T in terms of the temperature and Boltzmann constant? What is the average kinetic energy of translational motion of one of the molecules in terms of T and Boltzmann constant? **[5]**

Question A9

Express the isothermal bulk modulus, κ, in terms of thermodynamic variables and a partial differential involving those variables. Be clear which variable is being held constant. **[5]**

Question A10

An ideal gas is expanded in an isobaric process at pressure P from volume V to a volume $2V$. Write down the work done ***including sign*** and ***state whether this is work done on or by the gas***.

[5]

SECTION B. Answer two of the four questions in section B

B1) **a)**

i) Write down an expression for the incremental entropy change, dS, of a thermodynamic system as an amount of heat, δQ_R, flows to or from that system at temperature T. Make it clear how the sign of dS is determined. **[1]**

ii) Write down a simple expression of the Clausius inequality. **[1]**

b) 0.5kg of ice at 0 ^{o}C is mixed with with 6kg of water at 25 ^{0}C in an adiabatic container at 1 atmosphere.

i) Specify ***completely*** the final state of the system. **[3]**

ii) What is the net entropy change of the water-ice system, ΔS ? **[3]**

iii) Is the mixing process reversible? **[1]**

iv) State with reason whether the Second Law of Thermodynamics has been obeyed.

[2]

c) A mass of water, m, at temperature T_1 is mixed with an equal mass of water at temperature T_2 under isobaric and adiabatic conditions.

i) Demonstrate that the entropy change of the universe is given by;

$$\Delta S_{Uni} = 2mc_P \, ln\left[\frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}\right]$$ **[6]**

ii) Considering the above expression for the entropy change of the universe express the requirement that the Second Law is obeyed in terms of T_1 and T_2. **[4]**

iii) Demonstrate that the Second Law has been obeyed. **[4]**

Hint: Use the fact that $(a-b)^2 \geq 0$ ***for the case when a and b are both real numbers***

B2) **a)**

The Helmholtz free energy, F, for a fluid is defined as

$$F = U - TS$$

i) Use this definition and the thermodynamic identity to find the natural variables for F. **[2]**

ii) Show that $S = -\left(\frac{\partial F}{\partial T}\right)_V$ and that $P = -\left(\frac{\partial F}{\partial V}\right)_T$. **[3]**

iii) Use the information from parts i) and ii) to establish the Maxwell relation

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$ **[3]**

b)

i) Write down an expression for the heat capacity at constant volume, C_V, of a P-V system ***in terms of partial differentials involving only state variables*** and hence show that $C_V = \frac{3}{2} nR$ for a monatomic ideal gas. **[3]**

ii) Using the thermodynamic identity for a P-V system show that

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P$$ **[2]**

iii) Use the result from ii) with any necessary result from a) and the equation of state of the van der Waals gas,

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

to show that for such a gas

$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{n^2 a}{V^2}$$ **[6]**

iv) The heat capacity at constant volume of a van der Waals gas is the same as that for an ideal gas. Use this fact and the result from iii) to show that the internal energy of a van der Waals gas is given by

$$U = \frac{3}{2}nRT - \frac{n^2 a}{V} + const$$ **[6]**

B3)

a)

For a gas of molecules of mass m at temperature T the Maxwell speed distribution function is given by

$$P(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} 4\pi v^2 \exp\left[-\frac{mv^2}{2k_B T}\right].$$

i) The average speed of a molecule also known as the mean speed is related to the speed distribution function by

$$\bar{v} = \int_0^\infty vP(v)dv\,.$$

Evaluate this integral by parts and show that

$$\bar{v} = \sqrt{\frac{8k_B T}{\pi m}}\,.$$ **[5]**

Hint: Use the standard integral;

$$I_j = \int_0^\infty x^j \exp\left(-\alpha x^2\right)dx$$

with the standard results

$$I_0 = \sqrt{\frac{\pi}{4\alpha}}, \quad I_1 = \frac{1}{2\alpha}, \quad I_{2n} = \frac{2n-1}{2\alpha}I_{2n-2}, \quad I_{2n+1} = \frac{n}{\alpha}I_{2n-1}$$

The number of atoms colliding with or crossing unit area per unit time or flux, Φ, is given by $\Phi = \frac{1}{4}n\bar{v}$ where $n = \frac{N}{V}$ is the number density of atoms with N being the total number of atoms in a volume V.

ii) Using the ideal gas equation of state to calculate n, find number density, n_{O_2}, of Oxygen molecules in the air at room temperature (= 300 K) and atmospheric pressure where O_2 makes up 20% of the molecules in the atmosphere. **[3]**

Question continued overleaf

Turn Over

B3 Continued

iii) Given the mass of an O_2 molecule is 32 amu, find the number of O_2 molecules hitting 1 cm^2 of your lung per second. **[4]**

b)

If a first chamber containing a mixture of two gases is connected to a second chamber, under vacuum, by a hole whose diameter is small compared to the mean free path then the number of molecules escaping from the first to the second chamber via the hole will depend on the species. The process is known as effusion.

i) Using some of the information given in part a) show that the ratio of the number of species 1 molecule to species 2 that have escaped, $\frac{N_1^e}{N_2^e}$, is given by

$$\frac{N_1^e}{N_2^e} = \frac{n_1}{n_2}\sqrt{\frac{m_2}{m_1}}$$ **[3]**

ii) Suppose we have a mixture of 10% ^{3}He and 90% ^{4}He in the first chamber. What is the ratio of these isotopes that collects in the second chamber? **[4]**

iii) To produce a mixture with 30% ^{3}He we would need to repeat this procedure on the newly produced mixture and continue this recycling approximately how many times? **[6]**

B4)

a) An ideal gas is contained in a cylinder with a frictionless piston;

i) If the gas is expanded isobarically from pressure P_i and volume V_i to volume $2V_i$ what is the work done in this process and is it done by or on the gas? **[2]**

ii) If the gas is compressed adiabatically from pressure P_i and volume V_i to volume $\frac{V_i}{2}$ what is the work done in this process and is it done by or on the gas? **[2]**

iii) If the gas is compressed isothermally from pressure P_i and volume V_i to pressure $2P_i$ and volume $\frac{V_i}{2}$ what is the work done and is it done by or on the gas? **[2]**

b) A diesel engine cycle may be approximated by the cycle shown in figure 1.

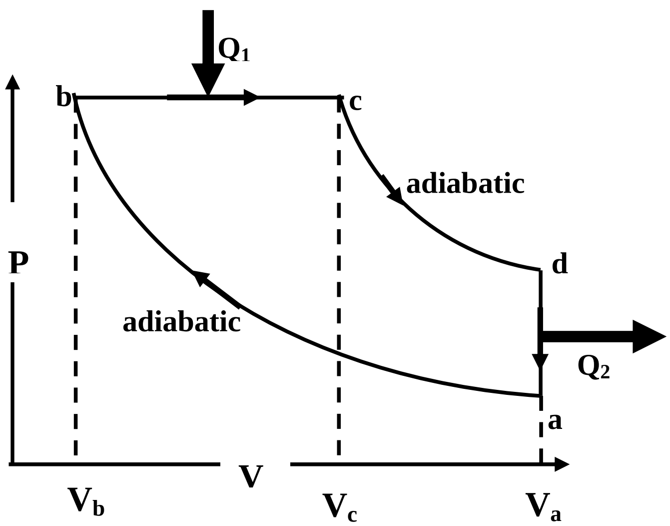

Figure 1

i) Find an expression for the heat absorbed by the system, Q_1, in the isobaric process $b \to c$ in terms of the ***relevant*** heat capacity and the temperatures T_c and T_b.

[2]

ii) Find an expression for the heat rejected by the system, Q_2, in the isochoric process $d \to a$ in terms of the ***relevant*** heat capacity and the temperatures T_d and T_a.
[2]

iii) Show that in terms of the temperatures the engine efficiency is

$$\eta_E = 1 - \frac{1}{\gamma}\frac{T_d - T_a}{T_c - T_b}.$$ **[2]**

Question continued overleaf

Turn Over

B4 continued

c)

i) We may define for the engine cycle of part b) an expansion ratio $\gamma_e = \frac{V_a}{V_c}$ and a compression ratio $\gamma_c = \frac{V_a}{V_b}$.

By considering the adiabats $c \to d$ and $a \to b$ show that the engine efficiency may be written in terms of the expansion and compression ratios as

$$\eta_E = 1 - \frac{1}{\gamma}\left[\frac{\gamma_e^{-\gamma} - \gamma_c^{-\gamma}}{\gamma_e^{-1} - \gamma_c^{-1}}\right]$$ **[5]**

ii) If the engine of part b) is designed to operate using a gas of rigid diatomic molecules with $V_a = 5000$ cm^3, $V_b = 500$ cm^3 and $V_c = 3000$ cm^3 , calculate the engine efficiency. **[2]**

iii) The engine described in part b) with the operating parameters of c) ii) is operated in reverse and used as a heat pump to warm a room. If the input power $\dot{W}$ is 3 kW how much heat per second is delivered to the room and how much heat is extracted from the cold reservoir per second? **[6]**

END OF PAPER

DATA SHEET

You may wish to use some of the following data.

c	= speed of light in vacuum	=	3×10^{8} m s^{-1}
k_B	= Boltzmann's constant	=	1.38×10^{-23} J K^{-1}
N_A	= Avagadro's number	=	6.02×10^{23} mol^{-1}
R	= Gas constant	=	8.31 J mol^{-1} K^{-1}
P_{atm}	= Atmospheric pressure	= 1atm =	1.01×10^{5} Pa
σ	= Stefan-Boltzmann constant	=	5.67×10^{-8} W m^{-2} K^{-4}
T_S	= Ice point of water	=	273.15 K
ρ_{Eth}	= Density of alcohol		0.789 gm.cc^{-1}
c_P	= Specific heat of water at constant pressure	=	4.2×10^{3} J K^{-1} kg^{-1}
c_{Eth}	= Specific heat of alcohol at constant pressure	=	2.4×10^{3} J K^{-1} kg^{-1}
c_P^{Ice}	= Specific heat of ice at constant pressure	=	2.1×10^{3} J kg^{-1}
l^{SL}	= Latent heat of melting ice	=	3.33×10^{5} J kg^{-1}
l^{LV}	= Latent heat of evaporating water	=	2.26×10^{6} J kg^{-1}
1 *amu*	= One atomic mass unit	=	1.66×10^{-27} kg