MSci 4242 Relativistic Waves & Quantum Fields

Problem Set 7

1. Prove that the Feynman propagator for a Dirac fermion

$$iS_F(x-y) = \langle 0|T\psi(x)\bar{\psi}(y)|0\rangle = -\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{\not{p}+m}{p^2-m^2+i\epsilon}$$

obeys the differential equation of a Green's function for the Dirac operator:

$$(i\not{\partial}_x - m)iS_F(x-y) = -\delta^{(4)}(x-y)\ .$$

2. Consider a real scalar field (or neutral Klein Gordon field) with creation operators $a^\dagger(\vec{p})$ and annihilation operators $a(\vec{p})$. Using the commutation relations for creation and annihilation operators and assuming that the vacuum state is normalised to one, i.e. $\langle 0|0\rangle = 1$, calculate the following expressions:

$$\langle 0|a(\vec{p}_2)a^\dagger(\vec{p}_1)|0\rangle$$

$$\langle 0|a(\vec{p}_4)a(\vec{p}_3)a^\dagger(\vec{p}_2)a^\dagger(\vec{p}_1)|0\rangle$$

Recall that $a^\dagger(\vec{p}_1)|0\rangle$ and $a^\dagger(\vec{p}_2)a^\dagger(\vec{p}_1)|0\rangle$ represent a one-particle and a two-particle state respectively, hence, what is the interpretation of the expressions above?

3. The Dirac field has the mode expansion

$$\psi(x) = \sum_{s=1,2} \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \left(b(\vec{p},s)U(p,s)e^{-ip\cdot x} + c^\dagger(\vec{p},s)V(p,s)e^{ip\cdot x} \right)\ .$$

Find the normal ordered charge operator $Q = \int d^3x \psi^\dagger \psi$ in terms of the creation and annihilation operators $b^\dagger, b, c^\dagger, c$, which obey anti-commutation relations introduced in the lecture. You may use the following identities:

$$U^\dagger(p,s)U(p,s') = V^\dagger(p,s)V(p,s') = 2E_{\vec{p}}\delta_{ss'}$$

and for $p^\mu = (E_{\vec{p}}, \vec{p})$ and $(p')^\mu = (E_{\vec{p}}, -\vec{p})$

$$U^\dagger(p',s)V(p,s) = V^\dagger(p',s)U(p,s) = 0\,.$$