UNIVERSITY OF LONDON

MSci EXAMINATION 2004

For Internal Students of

Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

PH4512A: NUCLEAR MAGNETIC RESONANCE

Time Allowed: **TWO AND A HALF** hours

Answer ***THREE QUESTIONS*** only

No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

Only CASIO fx85WA Calculators or CASIO fx85MS Calculators are permitted

PH4512A/21

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$	$H\ m^{-1}$
Permittivity of vacuum	ε_0	=	8.85×10^{-12}	$F\ m^{-1}$
	$1/4\pi\varepsilon_0$	=	9.0×10^{9}	$m\ F^{-1}$
Speed of light in vacuum	c	=	3.00×10^{8}	$m\ s^{-1}$
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	$C\ kg^{-1}$
Planck constant	h	=	6.63×10^{-34}	J s
	$\hbar = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	$J\ K^{-1}$
Stefan-Boltzmann constant	σ	=	5.67×10^{-8}	$W\ m^{-2}\ K^{-4}$
Gas constant	R	=	8.31	$J\ mol^{-1}\ K^{-1}$
Avogadro constant	N_A	=	6.02×10^{23}	mol^{-1}
Gravitational constant	G	=	6.67×10^{-11}	$N\ m^{2}\ kg^{-2}$
Acceleration due to gravity	g	=	9.81	$m\ s^{-2}$
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	m^{3}
One standard atmosphere	P_0	=	1.01×10^{5}	$N\ m^{-2}$

MATHEMATICAL CONSTANTS

$e \cong 2.718$ $\pi \cong 3.142$ $\log_e 10 \cong 2.303$

1. The equation of motion for the complex transverse magnetic moment

$$m = m_x + i m_y$$

may be written as

$$\frac{\mathrm{d}}{\mathrm{d}t} m(t) = -i\gamma \left\{ B_0 + b(t) \right\} m(t)$$

where B_0 is the static magnetic field in the z direction and $b(t)$ is a field which fluctuates because of the bodily motion of the magnetic moments.

(a) Discuss the origin of the field $b(t)$ in a typical system and explain the meaning of the other terms in the equation. **[5]**

(b) Show that the equation of motion has the formal solution

$$m(t) = m(0) e^{-i\gamma B_0 t} e^{-i\gamma \int_0^t b(\tau)\mathrm{d}\tau} .$$ **[5]**

(c) By taking an average over an ensemble of particles show that, under certain circumstances the mean transverse magnetisation will decay in an exponential manner. Explain clearly the assumptions you make. **[10]**

2. (a) The NMR relaxation times in a dipolar fluid may be written as

$$\frac{1}{T_1} = J_1(\omega_0) + 4J_2(2\omega_0)$$

$$\frac{1}{T_2} = \frac{3}{2} J_0(0) + \frac{5}{2} J_1(\omega_0) + J_2(2\omega_0).$$

Explain clearly the meaning of the various terms in the expressions and the physical ideas the equations contain. **[7]**

(b) Discuss how the various spectral density functions $J_n(\omega)$ depend on the motion of the nuclear spins and sketch the general behaviour of T_1 and T_2 as the temperature or the speed of the motion varies, explaining the features. **[13]**

PART MARKS

3. (a) Show that if a specimen of magnetic susceptibility

$$\chi(\omega) = \chi'(\omega) - i\chi''(\omega)$$

is placed inside a coil of inductance L, there appears to be an additional resistance r,

$$r = \omega L \chi'' \eta$$

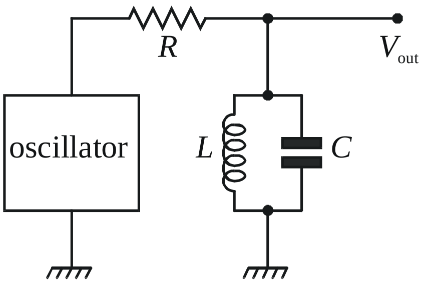
in series with the inductor. Here ω is the angular frequency. **[3]**

(b) What is the quantity η in the above equation? **[2]**

(c) Show that the Q factor of the coil is related to χ'' by

$$Q^{-1} = \chi'' \eta .$$ **[3]**

(d) Describe the operation of the Q-meter circuit:



and show how it may be used for the detection of NMR. **[12]**

4. (a) Draw a block diagram showing the components of a basic pulsed NMR spectrometer. **[5]**

(b) Sketch the form of typical signals at relevant parts of the spectrometer. **[5]**

(c) Explain the operation of the *gate* and the *mixer*. **[5]**

(d) Explain the purpose of the pulse programmer and describe pulse sequences for making simple measurements of T_1 and T_2. **[5]**

5. (a) Describe how spatial information may be encoded to frequency in NMR through the application of magnetic field gradients. **[4]**

(b) A magnetic field gradient $G_x = \partial B_z / \partial x$ is applied for a time t_x, followed by a gradient $G_y = \partial B_z / \partial y$ applied for time t_y, followed by a gradient $G_z = \partial B_z / \partial z$ applied for time t_z. Show that the relative phase accumulated by a spin at location **r** with coordinates x, y, z is

$$\varphi(\mathbf{r}) = \gamma\left(xG_x t_x + yG_y t_y + zG_z t_z \right).$$

[4]

(c) Show that the amplitude F of the NMR signal following the application of such gradient pulses may be written as

$$F(\mathbf{k}) = \iiint_{\substack{\text{volume of}\\ \text{specimen}}} \rho(\mathbf{r}) e^{i\mathbf{k}.\mathbf{r}} \mathrm{d}^3 r$$

where $\rho(\mathbf{r})$ is the density of spins at position **r**. Give the definition of the vector **k** in terms of other quantities defined in this question. **[4]**

(d) Using the above results, describe one way in which the spatial variation of the spin density in the specimen may be found. **[4]**

(e) Explain the difference between a slice-selective and a broad-band excitation pulse. **[4]**

END