UNIVERSITY OF LONDON

MSci EXAMINATION 2004

For Internal Students of

Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

PH4502B: LOW TEMPERATURE PHYSICS AND NANOTECHNOLOGY

Time Allowed: **TWO AND A HALF** hours

Answer ***THREE QUESTIONS*** only

No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

Only CASIO fx85WA Calculators or CASIO fx85MS Calculators are permitted

PH4502B/33

2003-04

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$	$H\ m^{-1}$
Permittivity of vacuum	ε_0	=	8.85×10^{-12}	$F\ m^{-1}$
	$1/4\pi\varepsilon_0$	=	9.0×10^{9}	$m\ F^{-1}$
Speed of light in vacuum	c	=	3.00×10^{8}	$m\ s^{-1}$
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	$C\ kg^{-1}$
Planck constant	h	=	6.63×10^{-34}	J s
	$\hbar = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	$J\ K^{-1}$
Stefan-Boltzmann constant	σ	=	5.67×10^{-8}	$W\ m^{-2}\ K^{-4}$
Gas constant	R	=	8.31	$J\ mol^{-1}\ K^{-1}$
Avogadro constant	N_A	=	6.02×10^{23}	mol^{-1}
Gravitational constant	G	=	6.67×10^{-11}	$N\ m^2\ kg^{-2}$
Acceleration due to gravity	g	=	9.81	$m\ s^{-2}$
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	m^3
One standard atmosphere	P_0	=	1.01×10^{5}	$N\ m^{-2}$

MATHEMATICAL CONSTANTS

$e \cong 2.718$ $\qquad \pi \cong 3.142$ $\qquad \log_e 10 \cong 2.303$

PART MARKS

Answer 3 questions only, including at least one question from section A and at least one from section B.

Section A

1. (a) Discuss fully the assumptions of the two fluid model of superfluid ^{4}He.

Apply the model to give an account of (i) the fountain pressure of superfluid ^{4}He and (ii) the Andronikashvilii experiment to measure the density and viscosity of the normal component. In each case carefully describe the experimental set-up, the observations, and how these are explained by the model. **[10]**

(b) Describe briefly the phenomenon of thermal counterflow in superfluid ^{4}He. Explain why the normal fluid velocity is given by the expression

$$\dot{q} = \frac{\dot{Q}}{A} = T\rho s v_n$$

where: $\dot{q} = \dot{Q}/A$ is the applied heat flux generated by a resistive heater dissipating power $\dot{Q}$ at one end of a column of fluid of area A; ρ, s and T are the total density, entropy per unit mass and temperature respectively.

From the data given below calculate the normal and superfluid velocities at 1.2, 1.8, and 2.1 K, when the applied heat flux is 10 W m^{-2}. **[4]**

Question 1 continued overleaf

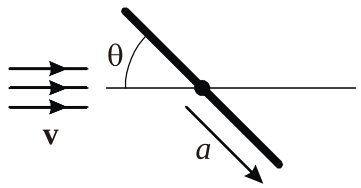
PART MARKS

Section A Continued

(c) In classical hydrodynamics it was shown by Lord Rayleigh that, when an inviscid fluid (i.e. a fluid assumed to have zero viscosity) flows past a circular disc, which is mounted on a torsion fibre normal to the fluid flow, and in the plane of the disc, the disc experiences a torque tending to align it perpendicular to the flow, given by

$$\tau = \frac{4}{3} a^3 \rho v^2 \sin 2\theta$$

Here θ is the angle between the flow velocity and the plane of the disc, a is the radius of the disc, and ρ is the density of the fluid.



Show that if such a disc is placed in superfluid ^{4}He, with $\theta = 45°$, it experiences a torque in the presence of a thermal counterflow given by **[4]**

$$\tau = \frac{4}{3} a^3 \frac{\dot{q}^2}{T^2 s^2 \rho} \frac{\rho_n}{\rho_s}$$

For the purposes of this calculation you should treat the normal component as inviscid as well.

(d) Discuss the temperature dependence of this torque. Describe briefly how the disc would respond to a second sound wave. **[2]**

Temperature (K)	Entropy (J kg^{-1} K^{-1})	Density (kg m^{-3})	Normal fraction (ρ_n/ρ)
1.2	52	145	0.029
1.8	535	145	0.32
2.1	1240	145	0.74

Section A Continued

2. (a) Discuss the phenomenon of Bose-Einstein Condensation (BEC) of (i) liquid ^{4}He modelled as a free ideal gas of bosons, and (ii) bosonic alkali gas atoms confined in a harmonic trap. **[6]**

(b) For a free ideal gas of N spinless massive bosons contained within a fixed volume V, show that a macroscopic population of the ground state implies that the chemical potential $\mu \approx 0$.

Hence show that the fraction of particles in the ground state varies with temperature as

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c}\right)^{3/2}$$

[7]

(c) Within this ideal Bose gas model, estimate the temperature dependence of the heat capacity at $T < T_c$. Sketch the form of the heat capacity as a function of temperature from $T = 0$ up to $T >> T_c$. **[3]**

(d) Discuss how the observed heat capacity of liquid ^{4}He differs from this result, both at the superfluid transition temperature, and at low temperatures.

Explain how the observed low temperature behaviour, at $T < T_c$, is accounted for by the elementary excitation spectrum of liquid ^{4}He. **[4]**

The Bose-Einstein distribution function is $\bar{n}(\varepsilon) = \dfrac{1}{\exp\left(\dfrac{\varepsilon - \mu}{k_B T}\right) - 1}$

The energy density of states is $g(\varepsilon) = \dfrac{2\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}$

Standard integral : $\displaystyle\int_0^\infty \frac{x^{1/2}dx}{e^x - 1} = 2.316$

Section A Continued

3. (a) Describe in detail the phase diagram of isotopic mixtures of liquid helium, and account for the maximum solubility of ^3He in liquid ^4He. **[6]**

(b) Describe, with the aid of a schematic diagram, the essential features of a ^3He-^4He dilution refrigerator, explaining the functions of the different components and its principles of operation. **[7]**

(c) Define the term "cooling power". Show that the cooling power developed at the mixing chamber is given by

$$\dot{Q} = 84 \dot{n}_3 T^2 \quad \text{(W)}$$

where $\dot{n}_3$ is the ^3He circulation rate in moles per second.

[The molar heat capacity of pure liquid ^3He = 22 T ($JK^{-1}mol^{-1}$); the heat capacity of saturated solution, per mole of ^3He = 106 T ($JK^{-1}mol^{-1}$)]

Contrast this expression with the cooling power of a pumped ^3He refrigerator. **[4]**

(e) An experimentalist wishes to detect gravitational waves, using as an antenna a 10 tonnes mass of copper cooled to 10 mK by a dilution refrigerator, with a ^3He circulation rate of 1000 μmole s^{-1}. Evaluate the claim that that the time taken to cool this mass from 1 K to 10 mK is less than one day.

You should assume that the molar heat capacity of copper arises solely from the electrons, with heat capacity $c = \gamma T$, where γ = 7 × 10^{-4} $JK^{-2}mol^{-1}$.

[1 tonne = 10^3 kg. The mass of one mole of copper is 0.0635 kg.] **[3]**

Section B

4. (a) What are the main tools of “top-down” and “bottom-up” nanotechnology? Give three examples of “bottom-up” nanotechnology techniques. **[4]**

(b) Describe briefly the key phenomenon taking place during the exposure of positive resist with an electron beam. Sketch the main steps of the positive resist process for patterning a thin film using electron beam lithography. **[5]**

(c) Explain how etching processes are used in nanolithography. What is the difference between isotropic and anisotropic etching processes? What is the degree of anisotropy? Give an example of an etching technique with a high degree of anisotropy. **[5]**

(d) Explain the mechanism for the proximity effect which occurs during electron beam lithography. Describe how corrections may be applied to minimise the results of the proximity effect. How may the choice of substrate minimise the effect? **[6]**

5. A certain conductor has the shape of a strip with the dimensions of

L_x = 1000 nm in the direction of the electrical current,

L_y =10 nm and L_z = 10 nm in the transverse directions.

The elastic scattering rate for the conduction electrons, $\tau^{-1} = 10^{14}$ s^{-1}; the phase breaking rate, τ_ϕ^{-1}, changes with temperature according to the law $\tau_\phi^{-1} = A\,T^3$ s^{-1}; $A = 2\times10^8$ s^{-1} K^{-3}; and the Fermi velocity, $v_F = 10^6$ m/s, the concentration of conduction electrons $n = 10^{28}$ m^{-3}, and the electron mass $m = m_e$, where m_e is the free electron mass.

(a) Calculate the temperature at which the conductor reaches the mesoscopic regime. **[6]**

(b) Using the Drude formula calculate the classical conductance of the conductor. **[5]**

(c) Estimate the relative amplitude of the Universal Conductance Fluctuations , $\Delta G/G$, for the conductor. **[5]**

(d) Describe the major electron scattering processes and their influence on quantum interference effects. **[4]**

PART MARKS

Section B Continued

6. (a) Describe the principles of operation of the Scanning Tunnelling Microscope (STM) in Constant Current Mode (CCI) and in Constant Height Mode (CHI). What are the practical limitations of CCI which can be overcome using CHI? **[4]**

(b) What are the main conditions for observation of the Coulomb blockade in single electron tunnelling in small junctions? **[5]**

(c) A certain tunnelling capacitor shows the Coulomb blockade at liquid helium temperature (4.2K). Calculate the transverse dimension of the capacitor for its plates in a shape of square with the thickness of insulating layer d = 2 nm made of Al_2O_3. What is the threshold voltage? ($\varepsilon = 2$ for Al_2O_3) **[6]**

(d) Describe a hanging bridge ("Dolan bridge") technique for the nanofabrication of nanometre scale tunnelling capacitors. **[5]**

END