Answer THREE questions.

The numbers in square brackets indicate the provisional allocations of maximum marks for each subsection of a question.

Vectors are denoted by bold-faced type, e.g., **a** and **B**, while scalar quantities, including the magnitude of the corresponding vector, are in italic type, e.g. q and $B = |\mathbf{B}|$.

You may assume, if needed, the following results:

- Maxwell's Equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla . \mathbf{E} = \frac{\rho_q}{\varepsilon_o}$$

$$\nabla \times \mathbf{B} = \mu_o \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \qquad \nabla . \mathbf{B} = 0$$

where **E** is the electric field vector, **B** the magnetic field vector, $c^2=(\varepsilon_o\mu_o)^{-1}$, ρ_q is the charge density, and **j** is the current density.

- Lorentz Force on single particle of charge q and velocity **v** is $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.

- MHD equations:

$$\frac{\partial \rho}{\partial \mathbf{t}} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad \rho\left(\frac{\partial}{\partial \mathbf{t}} + \mathbf{v} \cdot \nabla\right)\mathbf{v} = -\nabla \mathbf{P} + \mathbf{j} \times \mathbf{B}$$

$$\left(\frac{\partial}{\partial \mathbf{t}} + \mathbf{v} \cdot \nabla\right)\left(\mathbf{P}\rho^{-\gamma}\right) = 0 \qquad \mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta\, \mathbf{j}$$

For an electron - proton plasma, the mass density ρ is related to the number density n by $\rho = nm$ where $m = (m_i + m_e)$, **v** is the fluid flow velocity, P is the pressure, γ is the ratio of specific heats and η is the resistivity.

- Vector identity: $\nabla \times \nabla \times \mathbf{A} = \nabla (\nabla . \mathbf{A}) - \nabla^2 \mathbf{A}$

- Stokes Theorem: $\int_C \mathbf{B.dl} = \oint_S (\nabla \times \mathbf{B}).\mathbf{dS}$

- Physical constants:

Proton mass	$m_i = 1.67 \times 10^{-27}$ kg
Charge on an electron	$e = 1.6 \times 10^{-19}$ C
Vacuum dielectric constant	$\varepsilon_o = 8.85 \times 10^{-12}$ F m^{-1}
Boltzmann's constant	k $= 1.38 \times 10^{-23}$ J K^{-1}
Earth Radius	$R_E = 6.370 \times 10^6$ m
Equatorial surface magnetic field of Earth	$B_E = 31000$ nT

- Equation of a dipole magnetic field line is $r = r_{eq} \cos^2 \lambda$, where r is the radial distance from the planet centre, the subscript eq denotes the equatorial plane and λ is the magnetic latitude.

- The McIlwain parameter: $L = \dfrac{r_{eq}}{R_E}$

- The field strength at point (L, λ) in a dipole magnetic field:

$$B(L.\lambda) = \left(\frac{B_E}{L^3}\right)\left(\frac{\left[1 + 3\sin^2 \lambda\right]^{1/2}}{\cos^6 \lambda}\right)$$

Question 1.

a) The electric field drift velocity is given by

$$\mathbf{v_E} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

The magnetic field drift velocity in a dipole magnetic field is given by

$$\mathbf{v_M} = \left(W_\perp + 2W_\parallel\right)\left(\frac{\mathbf{B} \times \nabla B}{qB^3}\right)$$

where $W_\perp$ and $W_\parallel$ are the perpendicular and parallel particle energy respectively.

Consider a point at the magnetic equator on a field line of McIlwain parameter L. Show that the ratio of the magnetic field drift to the electric field drift, at this point, is given by

$$\left|\frac{\mathbf{v_M}}{\mathbf{v_E}}\right| = \frac{3\left(W_\perp + 2W_\parallel\right)}{qE_\perp L R_E}$$

[4 marks]

What is the energy in eV of a 90° pitch angle singly charged particle at $L = 6$, for which this ratio is equal to 1, if the combined convection and corotation electric field strength is 0.3 $\mathrm{mVm^{-1}}$ at $L = 6$?

[2 marks]

b) Write down an expression for the magnetic moment of a gyrating charged particle, and use it to find a relationship between the particle's pitch angle α and the local magnetic field strength in a time steady magnetic field.

[2 marks]

Explain briefly what is meant by the term "loss cone".

[2 marks]

Show that the loss cone angle in a dipole magnetic field can be written as

$$\sin^2 \alpha_{loss} = \frac{1}{\sqrt{4L^6 - 3L^5}}$$

[4 marks]

c) A spacecraft is on a magnetic field line of McIlwain parameter $L = 6$ and at a height above the surface of the Earth's northern hemisphere surface of 3180 km.

(i) It fires a beam of 100 keV electrons with a pitch angle of 160°. Calculate the equatorial pitch angle of the electrons and explain what happens to them next.

[2 marks]

(ii) It fires another beam of 100 keV electrons with a pitch angle of 95°. Again calculate their equatorial pitch angle and explain what happens to them next.

[2 marks]

(iii) Can either of these beams return to the spacecraft?

[2 marks]

Question 2.

a) Write down the definition of what makes a "collisionless plasma" more than simply an ionized gas, and explain the two key concepts.

[4 marks]

b) Describe what happens when an electrically charged object is placed in a plasma.

[4 marks]

c) The Debye length in an electron-proton plasma is given by the expression

$$\lambda_D{}^2 = \frac{\varepsilon_0 k_B T_e}{n_e q^2}$$

where the electron density and temperature are given by n_e and T_e respectively, and q is the electronic charge. Explain how the constraints associated with the two key concepts mentioned above in (a) can be characterised simply in terms of expressions involving the Debye length.

[**4 marks]**

d) The Cluster spacecraft are shaped like drums 3 m in diameter from which four 50 m long wire booms extend, equi-spaced around the perimeter. At the end of each wire boom is a probe which is electrically insulated from the rest of the spacecraft. In sunlight, the spacecraft body emits large numbers of photo-electrons, and becomes positively charged. The probes are electrically in contact with the plasma surrounding them. For the cases of

(i) a solar wind plasma of density 10^7 m^{-3} and temperature 5 eV, and
(ii) a ring current plasma of density10^6 m^{-3} and temperature 3000 eV,

determine whether more or less than 99% of the spacecraft potential is shielded in the plasma at the distance of the probes?

[**4 marks]**

e) Discuss what changes will occur to the potential of the spacecraft and to the potential measured at the probes if the spacecraft moves into the Earth's shadow while remaining in a plasma with a particular temperature and density.

[**4 marks]**

Question 3.

a) Show that the evolution of the magnetic field in MHD is governed by:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\eta}{\mu_o} \nabla^2 \mathbf{B}$$

where η is the resistivity of the plasma, assumed to be constant.

[4 Marks]

b) Describe the importance of each of the terms on the right hand side of the equation. Under which space plasma conditions might each of these terms be considered relevant?

[4 Marks]

c) Explain the significance of the *frozen-in flow* approximation. Draw diagrams and discuss how this concept can be used to understand the spiral form of the interplanetary magnetic field (IMF).

[4 Marks]

d) The rotation of the Sun causes the interplanetary magnetic field to become wound up into the Parker spiral configuration, since the frame co-rotating with the Sun has an additional azimuthal component $u_\phi = -\Omega r$. In a spherically symmetric model, the frozen-in flux condition implies that

$$\frac{B_\phi}{B_r} = \frac{u_\phi}{u_r} = -\frac{\Omega r}{u_r}$$

Use Maxwell's equations to determine the variation of the radial component of the magnetic field, B_r, with radial distance r, and thus use the above condition to determine the azimuthal component, B_ϕ, as a function of r.

[3 Marks]

e) Comment briefly on why this model of the interplanetary magnetic field structure may not be valid at very short distances from the Sun.

[2 Marks]

f) Spacecraft in near-Earth orbits observe $|B_r| = |B_\phi| \sim 5$ nT on average. What is the average field strength of the 2 components, and thus the Parker spiral angle, of the interplanetary magnetic field expected to be observed by the Cassini spacecraft as it enters Saturn orbit in July 2004? (Assume that $u_r \sim$ constant for the solar wind between the Earth and Saturn. Saturn is at a distance of 9.54 AU from the Sun.)

[3 Marks]

Question 4.

a) Draw a large, clearly labelled diagram illustrating the plasma and magnetic field boundaries you would expect to find in the region of space immediately upstream of the Earth or other magnetized planet. Briefly indicate the primary characteristics of the plasma populations outside and between these boundaries, and, where appropriate, how the structure of the boundaries are influenced by the direction of the interplanetary magnetic field.

[7 Marks]

b) A simple model of the current flowing in the magnetopause boundary layer can be constructed considering the motion of ions and electrons at the interface between a cold, flowing plasma with no magnetic field and a region of magnetic field devoid of plasma (the so-called Chapman-Ferraro model). Sketch the motion of ions and electrons within such an interface, and hence show that the balance of pressures across the boundary can be expressed as

$$nm_p v^2 = \frac{B^2}{2\mu_o}$$

where n is the upstream plasma number density, v is the bulk flow speed, m_p is the proton mass and B is the downstream magnetic field strength.

[6 Marks]

c) Describe the mechanisms by which particles may be accelerated to very high energies at or near the bow shock surface. How do these mechanisms relate to the large-scale structure of the associated shock?

[3 Marks]

d) The coplanarity theorem for magnetohydrodynamic shocks states that the magnetic field vectors upstream and downstream of the shock and the normal vector to the shock surface should all lie in the same plane. Use this theorem, with any appropriate shock jump conditions for the magnetic field, to determine the normal vector to a shock crossed by a spacecraft which observes an upstream magnetic field of (0.39, –3.32, –4.8) nT, and a downstream magnetic field of (–0.96, –5.65, –9.6) nT. What type of shock do you expect this to be?

[4 Marks]

Question 5. Produce brief notes for any FOUR of the following five topics:

i) Particle populations found in the inner magnetosphere ($L < 7$).

ii) The differences between the motion of energetic trapped charged particles in a simple model dipole magnetic field and in the non-dipolar terrestrial magnetic field in the inner magnetosphere ($L < 7$).

iii) The coupling of the magnetospheric convection cycle into the polar ionosphere.

iv) The processes occurring in and around the magnetosphere during magnetic storms.

v) The process by which oxygen atoms escaping from the atmosphere of Venus may be picked-up by the solar wind.

[5 Marks Per Topic]

 END OF PAPER