Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Permittivity of free space ε_0 = 8.85 x 10^{-12} F m^{-1}

Electronic charge: e = 1.6 x 10^{-19} C

Permeability of free space: $\mu_0 = 4\pi$ x 10^{-7} Hm^{-1}

Proton rest mass: m_p = 1.67 x 10^{-27} kg

Electron rest mass: m_e = 9.11 x 10^{-31} kg

Bohr radius: a_0 = 5.29 x 10^{-11} m

1. What states of matter constitute a plasma? [2]

A cold plasma of electron number density n_e is subjected to a small external perturbation causing a charge separation x. Discuss the motion of the electron and derive the expression for the electron plasma frequency

$$\omega_{pe} = \left[\frac{n_e e^2}{m_e \varepsilon_0}\right]^{1/2} .$$

Why can you neglect the ionic motion? [10]

The electron number density in the Solar corona is 10^{14} m^{-3}. Consider a volume element of radius 10^5 m and determine the charge imbalance within the volume required to produce a field of 50 Vm^{-1} at the spherical boundary. [4]

What is the maximum energy that a singly charged particle would acquire in the volume element? [4]

2. Define the electron cyclotron frequency ω_{ce}. [1]

An electron of velocity **V** interacts with a uniform magnetic field $\mathbf{B}_0$ which is parallel to the z-axis.

i Write down the Lorentz equations. [2]

ii Show that $\frac{d^2}{dt^2}\mathbf{V}_\perp = -\omega_{ce}^2\,\mathbf{V}_\perp$ where $\mathbf{V}_\perp = \mathbf{V}_x + \mathbf{V}_y$. [3]

iii Give the solutions for above equations for the positions x,y and z. [3]

You may assume the solutions of a simple harmonic equation.

iv Now consider the magnetic field as increasing along the y-axis as

$$\mathbf{B} = \left(B_0 + y\frac{\partial B_z}{\partial y}\right)\mathbf{e}_z$$

and show that an average additional acceleration $V_\perp^2 / 2\ell_c$ arises along the y-direction where $\ell_c = \left[\frac{1}{B_0}\frac{\partial B_z}{\partial y}\right]^{-1}$. [8]

v What will be the effect of the additional acceleration $V_\perp{}^2/2\ell_c$ on the electron's motion? [3]

3. The wave equation for the propagation of an electro-magnetic wave of frequency ω and amplitude **E,**

$$\nabla^2 \mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) + \frac{\omega^2}{c^2} \boldsymbol{\varepsilon} \cdot \mathbf{E} = 0$$

can be written in the operator form **M**·**E** = 0

i Assuming a cold magnetic plasma give an expression for **M** in terms of the dielectric tensor, **ε,** and the wave vector , **k.** Determine the matrix of **M** in terms of the components of the refractive index and the elements of the dielectric tensor. (See below.) [6]

ii Normal modes propagate in the cold plasma parallel to the magnetic field, *B*. When *B* is parallel to the *z*-axis and ε_3 is finite show that the magnitude of the wave vector, **k,** for right circularly polarized waves can be written as

$$k = \frac{\omega}{\mathrm{c}} \left[1 + \frac{\omega_{pe}^2}{\omega(\omega_{ce} - \omega)} \right]^{\frac{1}{2}}, \qquad [6]$$

where ω_{pe} and ω_{ce} are the plasma and cyclotron frequencies respectively.

iii Sketch a dispersion plot of these waves and identify the cut-off frequency, the resonance, the right circularly polarized wave and the Whistler wave. Derive an expression for the Whistler frequency in terms of ω_{pe} stating any approximations used. [8]

You may assume the dielectric tensor takes the form

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix},$$

where $\varepsilon_1 = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2 - \omega^2}$, $\varepsilon_2 = \frac{\omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega_{ce}^2 - \omega^2}$, $\varepsilon_3 = 1 - \frac{\omega_{pc}^2}{\omega^2}$.

4. The Alfen velocity V_A is given by

$$V_A = \left[\frac{B^2}{\mu_o n_1 M} \right]^{1/2}$$

where B is the magnetic field, M is the ion mass and n_i is the ion number density. Express V_A in terms of the plasma and cyclotron frequencies of the ions. [2]

Explain the justification for using the fluid description of a plasma. [4]

When considering the plasma as a conducting fluid, the force experienced by the fluid is given by

$$-\nabla \left[P + \frac{B^2}{2\mu_o} \right] + \frac{1}{\mu_o} [\mathbf{B} \cdot \nabla] \mathbf{B},$$

where P is the pressure.

Recast this expression in terms of the radius of curvature of the B field and distinguish between pressure and stress forces. [8]

A positron gun fires a 1 keV beam of positrons with a current of 1 Ampère and cross sectional area 10^{-4} m^2 at a cylindrical rocket ship. The rocket ship is shielded by a magnetic field parallel to its cylindrical axis of magnitude $B = B_0 \exp(-ar)$, where $a = 1$ m^{-1} and $B_0 = [4\pi]^{1/2}$ T. The positron beam is orthogonal to the z-axis. At what radial distance from the rocket ship will the positron beam be stopped? [6]

5. Consider an electron of velocity **V** moving through helium gas in the presence of a laser field of maximum amplitude $\mathbf{E}_0$ and frequency ω.

Distinguish between **collision frequency**, ν, and **collision duration,** τ. [2]

Determine ν when the helium number density is 3.5 x 10^{25} m^{-3} and the electron velocity in 6 x 10^5 m s^{-1}. Take the total elastic cross section for electron helium collisions to be $10\,\pi a_o^2$.
Estimate τ. [4]

Show that the classical energy of oscillation ε_c of the electron is

$\varepsilon_c = \dfrac{e^2 E_0^2}{4 m_e \omega^2}$ and state any restrictions that apply in the derivation. [3]

Given that the momentum $m_e\mathbf{V}$ of the electron is

$$m_{\mathbf{e}}\mathbf{V} = - \, i\, e\, E_o (\omega + i\,\nu)^{-1},$$

show that the average increase $\Delta\varepsilon$ in electron energy per collision is

$$\Delta\varepsilon = 2\,\varepsilon_c\,\omega^2\,(\omega^2 + \nu^2)^{-1}$$ [4]

Discuss the dependence of $\Delta\varepsilon$ over the whole range of ν values. [3]

By considering the Joule heat show that the conductivity, σ, in the high frequency laser field is given by

$$\sigma = \frac{e^2 n_e \nu}{m_e \omega^2},$$

where n_e is the election density. [4]