Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

[Part marks]

1. (a) Consider the QED elastic scattering reaction $e^-\mu^- \to e^-\mu^-$. Neglecting all masses, the cross section is given by

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{s}\frac{1+\cos^4(\theta/2)}{\sin^4(\theta/2)},$$

where α is the fine structure constant, s is the square of the centre-of-mass energy and θ is the angle of the scattered electron in the $e\mu$ centre-of-mass frame. Draw the Feynman diagram for this reaction and justify the power of α in the above expression. [3]

(b) Neglecting all masses in the above reaction, show that the change of the electron four-momentum, q^μ, satisfies

$$q^2 = q_\mu q^\mu = -\frac{1}{2}s(1-\cos\theta).$$

Hence show that the cross section as a function of q^2 is

$$\frac{d\sigma}{d(q^2)} = \frac{2\pi\alpha^2}{q^4}\left[\frac{q^4}{s^2} + 2\left(1+\frac{q^2}{s}\right)\right].$$

[6]

(c) Deep inelastic scattering of electrons from protons can be described by the reaction $e^-p \to e^-X$, where X is a hadronic system with $m_X \gg m_p$. Within the quark model, this can be considered to arise from elastic electron-quark scattering. If the scattered quark initially had a fraction x of the proton momentum then, neglecting masses and transverse momentum components, show that the centre-of-mass energy of the electron-quark system, $\hat{s}$, is related to that of the electron-proton system, s, by

$$\hat{s} = xs.$$

[4]

(d) If the probability of each type of quark i ($i = u,\ d,\ s,\ ...$) having a momentum x is $p_i(x)dx$, then using the results from (b) and (c), write down the cross section as a function of q^2 and x for the electron-proton reaction from the quark model. Conventionally, the cross section for the electron-proton reaction has been written as

$$\frac{d\sigma}{dx\,d(q^2)} = \frac{2\pi\alpha^2}{q^4}\left[\left(\frac{q^2}{xs}\right)^2 F_1(x,q^2) + \left(1+\frac{q^2}{xs}\right)\frac{F_2(x,q^2)}{x}\right]$$

in terms of two arbitrary "structure functions" F_1 and F_2. Hence deduce a relation between F_1 and F_2. [7]

2. (a) The main decay mode of the muon is $\mu^- \to \nu_\mu e^- \bar{\nu}_e$. Draw the lowest order Feynman diagram for this decay and explain why this decay can be approximated by a zero-range four-fermion interaction. [3]

(b) Neglecting the electron mass, what is the range of values that the energy E_e of the electrons emitted in this decay can take, in the muon's rest frame? [2]

(c) Neglecting the electron mass, the spectrum of E_e is given by

$$\frac{d\Gamma}{dE_e} = \frac{2G_F^2 m_\mu^2 E_e^2}{(2\pi)^3}\left(1 - \frac{4E_e}{3m_\mu}\right)$$

By differentiating this expression, find the most probable energy for the electron and sketch the energy spectrum over the full allowed range of E_e. [4]

(d) Draw a diagram showing the orientation of the momenta of the three outgoing particles in the muon's rest frame, when E_e is at its most probable value. For each particle, show the direction of its spin and give its helicity. Also show the direction of the muon spin. Briefly explain your answers. [6]

(e) Integrate the energy spectrum to obtain an expression for the total decay width of the muon. Hence, calculate the muon lifetime in seconds. The muon mass is 105.7 MeV the value of the Fermi constant is $G_F = 1.166 \times 10^{-5}$ GeV^{-2} and in natural units 1 second $\equiv 1.519 \times 10^{24}$ GeV^{-1}. [5]

3. The Higgs mechanism allows gauge bosons to have mass without violating local gauge invariance. In the Standard Model, the $W^{\pm}$ and the Z^0 acquire mass through this method, but it could be applied equally well to give mass to the photon.

   (a) The Lagrangian density for a photon with mass m_γ would be

   $$\mathcal{L}_\gamma = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\gamma^2 A_\mu A^\mu \,,$$

   where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Show explicitly that this Lagrangian density is not invariant under a gauge transformation

   $$A^\mu \to A^\mu + \partial^\mu \Lambda$$

   unless the mass is zero. [3]

   (b) Given the locally gauge invariant Lagrangian density

   $$\mathcal{L} = \tfrac{1}{2}\left[(\partial_\mu - iqA_\mu)\,\phi^\star\right]\left[(\partial^\mu + iqA^\mu)\,\phi\right] + \tfrac{1}{2}\mu^2|\phi|^2 - \frac{1}{4}\lambda^2|\phi|^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

   ($\mu, \lambda > 0$) find the minimum v (ground state), of the "potential" term for the ϕ field

   $$V(|\phi|) = -\tfrac{1}{2}\mu^2|\phi|^2 + \frac{1}{4}\lambda^2|\phi|^4$$

   [2]

   (c) Assuming a gauge has been chosen that eliminates the imaginary component of the ϕ field, express ϕ in terms of a new field H representing the deviations of ϕ from the ground state v. Then express $\mathcal{L}$ in terms of H. [7]

   (d) Identify the mass term for the photon and give an expression for the generated photon mass in terms of μ and λ. [4]

   (e) Identify any interaction terms and draw the corresponding Feynman diagrams. [4]

4. The CKM unitary matrix gives the flavour-dependent relative couplings for the charged-current weak interactions for quarks, where V_{ij} is the factor for interactions involving quarks i and j. The numerical values of the magnitudes of the matrix elements can be taken to be

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.975 & 0.223 & 0.003 \\ 0.222 & 0.974 & 0.040 \\ 0.009 & 0.039 & 0.999 \end{pmatrix}$$

(a) The tau lepton has a mass of 1.78 GeV and a lifetime of 0.29×10^{-12}s. It can decay semi-hadronically: to either a tau neutrino and one or more pions; or to a tau neutrino, a kaon and zero or more pions. Examples of these two types of decay are $\tau^- \to \nu_\tau \pi^-$ and $\tau^- \to \nu_\tau K^-$, respectively. Draw a quark level Feynman diagram for each of these particular example decays. Ignoring mass effects, estimate their relative rates. [5]

(b) The only other decays of the tau are leptonic: $\tau^- \to \nu_\tau e^- \bar{\nu}_e$ or $\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$. Ignoring mass effects, estimate the branching fractions for both of these decays and also for the two types of semi-hadronic decays described in part (a). [6]

(c) The charmed meson D^+ (quark content $c\bar{d}$) has a measured lifetime of 1.05×10^{-12}s. Draw a quark level Feynman diagram for the most common hadronic decay of this meson. [4]

(d) The total width of the tau is proportional to m_τ^5. Using this, and assuming asymptotic freedom holds for the D^+ decay ("spectator model"), estimate the D^+ lifetime and compare with the above value. The mass of the charm quark can be taken to be 1.4 GeV. [5]

5. (a) Describe briefly the reactions studied and the results of the SNO experiment. [6]

(b) Explain briefly the process of neutrino-less double beta decay and its significance in understanding the properties of neutrinos. [6]

(c) If neutrinos have mass and lepton number is not conserved, then the neutrino flavour eigenstates, $(\nu_e, \nu_\mu, \nu_\tau)$, need not be the same as the mass eigenstates, (ν_1, ν_2, ν_3), and mixing can occur. Consider the case with mixing only between the electron and muon states, i.e.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} .$$

A pure electron neutrino beam of momentum p is created at time $t = 0$. What is the composition of the neutrino state at a later time t? Show that the amplitude of muon neutrino production in the beam is given by

$$A_\mu = \cos\theta \sin\theta (e^{-iE_2 t} - e^{-iE_1 t}) ,$$

where E_i is the energy of the state ν_i. [5]

(d) Hence, show that the proportion of muon neutrinos in the beam after time t is

$$P_\mu = \sin^2 2\theta \sin^2 \left(\frac{(E_2 - E_1)t}{2} \right)$$

and explain how this can be used to extract information about the masses of ν_1 and ν_2, assuming that $E_i \gg m_i$. [3]

END OF PAPER