UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

M.Sci.

Physics 4442: Particle Physics

COURSE CODE : **PHYS4442**

UNIT VALUE : **0.50**

DATE : **12-MAY-05**

TIME : **14.30**

TIME ALLOWED : **2 Hours 30 Minutes**

05-C1102-3-40

TURN OVER

Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

[Part marks]

1. The running of the QCD coupling constant α_s is given as a function of the energy scale Q^2 by the expression

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(11n_c - 2n_f)\log(Q^2/\mu^2)}$$

where n_c is the number of colours, n_f is the number of quark flavours, and μ^2 a reference energy scale.

(a) Based on the above formula explain briefly the behaviour of the strong interactions at low and high energies. [4]

(b) How do we know experimentally the number of colours? [3]

(c) How do we know experimentally that quarks have spin $\frac{1}{2}$? [3]

(d) How do we know experimentally that gluons have spin 1? [3]

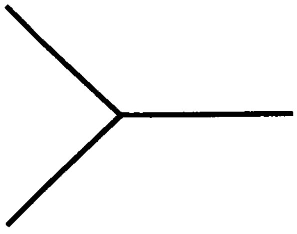
(e) How do we know experimentally that gluons carry colour? [3]

(f) In electron-proton (or electron-neutron) inelastic scattering, the structure function $F_2(x, Q^2)$ is defined as

$$F_2(x, Q^2) = \sum_q (\frac{e_q}{e})^2 f_q(x, Q^2)$$

where $f_q(x, Q^2)$ is the momentum density for a quark of flavour q and the sum runs over all quark flavours. What would the ratio $F_2^{proton}/F_2^{neutron}$ tend to for $x \to 0$? Briefly explain your answer. [4]

2. An imaginary world consists of just three types of particles: A, B and C. They all have spin 0 and each is its own antiparticle. There is only one vertex by which the particles interact:



and the strength of the interaction is determined by a coupling constant g.

(a) Draw the lowest order Feynman diagram(s) and determine the amplitude, $\mathcal{M}$, for the process $A + A \rightarrow B + B$. Express $\mathcal{M}$ in terms of the Mandelstam variables. [7]

(b) The differential cross section for a two-body scattering process in the centre-of-mass (CM) frame is given by Fermi's Golden Rule:

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2} \frac{|\mathcal{M}|^2}{E_{CM}^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

where $|\vec{p}_i|$ and $|\vec{p}_f|$ are the initial and final state momenta. Using this, and assuming that $m_A = m_B$ and $m_C = 0$, find the differential cross section for $A + A \rightarrow B + B$ in the CM frame and express it in terms of the CM energy, E_{CM}, and the scattering angle, θ. You may assume that E_{CM} is high enough that approximations such as $E_A \approx |\vec{p}_A|$ can be made.
Assuming that $m_A = m_B$ and $m_C = 0$, find the differential cross section for $A + A \rightarrow B + B$ in the centre-of-mass (CM) frame and express it in terms of the CM energy, E_{CM}, and the scattering angle, θ. You may assume that E_{CM} is high enough that approximations such as $E_A \approx |\vec{p}_A|$ can be made. Also, you may use Fermi's Golden Rule for a two body scattering in the CM frame: [7]

(c) Draw the lowest order Feynman diagrams for the process $A + A \rightarrow A + A$ and give a qualitative estimate of its relative rate with respect to $A + A \rightarrow B + B$, based on the number of vertices in the diagrams. [6]

3. The charged pion, with spin zero and mass 139.6 MeV, can decay to an electron (mass 0.511 MeV) or a muon (mass 105.7 MeV), through the decay

$$\pi^- \to \ell^- \bar{\nu}_\ell ,$$

where ℓ stands for e or μ. These decays have branching fractions of 1.23×10^{-4} and 0.99988 respectively. Any neutrino mass should be neglected in the following

(a) Draw a Feynman diagram for this decay. [3]

(b) According to Fermi's Golden Rule, the partial width Γ_i for a particle of mass m to decay to a mode i is

$$\Gamma_i = \frac{|\mathcal{M}_i|^2 \rho_i}{2m} ,$$

where $\mathcal{M}_i$ is the amplitude and ρ_i is the Lorentz invariant phase space. Briefly explain the physical significance of $\mathcal{M}_i$, ρ_i and Γ_i. [3]

(c) The phase space available for the above pion decays is

$$\rho_\ell = \frac{1}{8\pi} \frac{m_\pi^2 - m_\ell^2}{m_\pi^2} .$$

Evaluate the ratio of the magnitudes of the phase space factors for those two decays and comment on your result. [2]

(d) The lowest order amplitude for these decays is

$$|\mathcal{M}_i|^2 = 2G_F^2 f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2) ,$$

where G_F is the Fermi constant and f_π the pion form factor. Calculate an expression for the ratio of the partial widths of these two decays. Evaluate this ratio and comment on your result. [3]

(e) A left-handed state of a fermion has components of both helicity $\pm 1/2$ states with amplitude $\sqrt{(1 \mp \beta)/2}$ respectively, where β is the velocity of the particle. Given that the weak interactions couple only to left-handed particles (and hence right-handed antiparticles) draw a diagram showing the lepton and antineutrino helicities in these decays and explain qualitatively the form of $|\mathcal{M}_i|^2$. [5]

(f) The charged kaon has a mass of 493.7 MeV. The branching fraction for the equivalent muon decay, $K^- \to \mu^- \bar{\nu}_\mu$, is 0.6351. Briefly explain why this is smaller than for the pion case and estimate the branching fraction for the decay $K^- \to e^- \bar{\nu}_e$. [4]

4. The standard representation of the free Dirac particle solution is

$$\psi = \sqrt{(E+m)} \begin{pmatrix} \chi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi \end{pmatrix} e^{-ip_\mu x^\mu}$$

and the standard representation of the γ matrices (fundamental relation $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$) is

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

(a) How does ψ change under the Parity transformation, $\hat{P}$? Show that this can be written as

$$\hat{P}\psi = \gamma^0\psi\,.$$

(This is a general result, independent of the representation for the Dirac spinors and the γ matrices.) [4]

(b) Show that the free Dirac Lagrangian density

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

is invariant under the Parity transformation, while replacing γ^μ with $\gamma^\mu(1-\gamma^5)$ breaks the $\hat{P}$ invariance. Comment on this result. [5]

(c) Show that the operators

$$P_R \equiv \tfrac{1}{2}(1+\gamma^5) \qquad P_L \equiv \tfrac{1}{2}(1-\gamma^5)$$

have the appropriate properties to be (right- and left-hand) projection operators, that is

$$P_i^2 = P_i, \qquad P_L + P_R = 1, \qquad P_R P_L = 0\,.$$

[3]

(d) Show that if $m << E$

$$\gamma^5\psi \approx \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix}\psi\,,$$

where $\hat{p} = \vec{p}/|\vec{p}|$. [4]

(e) Hence show that in the ultra-relativistic limit the projection operators $P_{R,L}$ project out the positive and negative helicity components of a free Dirac spinor. [4]

5. The CKM unitary matrix gives the flavour-dependent relative couplings for the charged-current weak interactions for quarks, where V_{ij} is the factor for interactions involving quarks i and j. The numerical values of the magnitudes of the matrix elements can be taken to be

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.975 & 0.223 & 0.003 \\ 0.222 & 0.974 & 0.040 \\ 0.009 & 0.039 & 0.999 \end{pmatrix}$$

(a) Explain the large suppression of the $\mu^+\mu^-$ decay mode of neutral kaons. Why is this decay not totally suppressed? [4]

(b) List all the possible decay modes of the W^- boson. [3]

(c) Draw the lowest order Feynman diagrams for the process $e^+e^- \rightarrow W^+W^-$. [4]

(d) What is the probability for both W bosons in an $e^+e^- \rightarrow W^+W^-$ event to decay into quarks? [6]

(e) The decay lifetime of hadrons containing a b or a c quark essentially reflects the lifetime of the heavy quark itself, which proceeds via the weak interactions. Given that the b quark is heavier than the c quark, on the grounds of phase space, one would expect the b hadrons lifetime to be much shorter than the that of c hadrons. Experimentally we find the opposite. Draw the main Feynman diagram for the decay of b and c and explain briefly the above observation. [3]

END OF PAPER