

## SSP Exercise 2

To be handed in by 4pm, Thursday 26 January.

For a simple cubic lattice the reflections with the following Miller indices are possible: (100), (110), (111), (200), (210), (211), (220), (etc.). An x-ray diffraction experiment with the wavelength of  $\lambda = 1\text{\AA}$  reveals a set of peaks recorded at the following angles  $\vartheta$  (deg.):

5.74
8.13
9.98
11.54
12.93
14.19
16.44

- a. Show that this x-ray diffraction picture corresponds to a simple cubic lattice. **Hint**, note that the sequence  $n$  formed by  $n = h^2 + k^2 + l^2$  for a simple cubic lattice is 1,2,3,4,5,6,8. Rewrite the Bragg law in the form

$$\sin^2 \theta = \frac{\lambda^2}{4d^2} \quad \text{Eq. 1}$$

and express  $d$  via  $a$  and  $h, k, l$ . Can you obtain the sequence  $n$  above from the recorded peaks using Eq.1? **[10]**

- b. Calculate the value of the lattice constant  $a$ . **[5]**

### Solution:

- a. From Eq.1 the Bragg law can be rewritten as

$$\sin^2 \theta = \frac{\lambda^2}{4a^2} * n = C * n, \text{ where we use } a = \frac{d}{\sqrt{h^2 + k^2 + l^2}}$$

Now if we use the sequence of angles above in the form

$$\sin^2 \theta_1 = C * n_1$$

$$\sin^2 \theta_2 = C * n_2$$

$$\sin^2 \theta_3 = C * n_3$$

...

$$\sin^2 \theta_8 = C * n_8$$

and assuming that  $n_1=1$  divide as follows

$$\frac{\sin^2 \theta_2}{\sin^2 \theta_1} = \frac{n_2}{n_1} \approx \frac{0.02}{0.01} = n_2 = 2$$

$$\frac{\sin^2 \theta_8}{\sin^2 \theta_1} \approx \frac{0.08}{0.01} = 8$$

To recover the sequence 1, 2,3,4,5,6,8.

b. Now use the Bragg law in the form:

$$\sin^2\theta = \frac{\lambda^2}{4a^2}(h^2 + k^2 + l^2)$$

to obtain

$$a = \frac{\lambda}{2\sin\theta}(\sqrt{h^2 + k^2 + l^2})$$

and for (100) we have  $a = \frac{\lambda}{2\sin\theta_1} = \frac{1}{2*0.1} = 5\text{\AA} = 5 * 10^{-10}m$