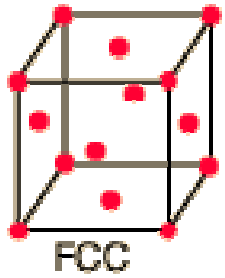
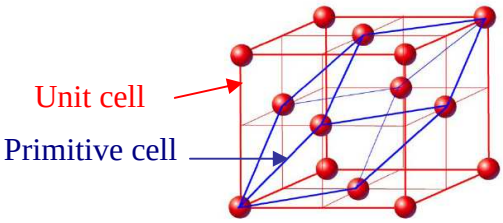
PHY-550 Solid State Physics
2010 Exam Solutions

Section A

A1. i)



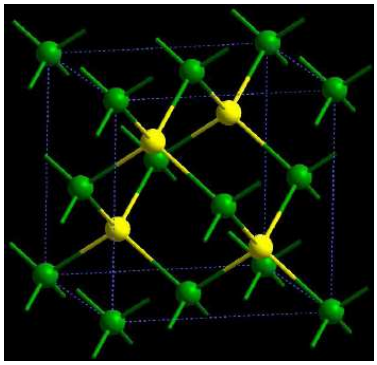
ii)



[2]

4 lattice points per unit cell. **[2]**

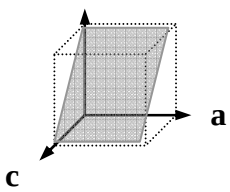
A2.



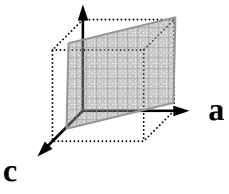
Green Ga, yellow As **[4]**

A3.

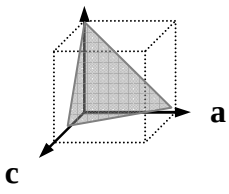
(011) **b** **[1]**



(102) **b** **[1]**



(211) **b** **[1]**



A4. $\varepsilon(\mathbf{k}) = \dfrac{\hbar^2 k^2}{2m}$, k=wavevector, m=mass **[2]**

A5. Total ground state electrons = $2.\left(\frac{4\pi k_F^3}{3}\right)\left(\frac{V}{8\pi^3}\right) = \frac{k_F^3}{6\pi^2}V$

Two spin orientations

k-space volume, k_F=Fermi wavevector

Hence electron density, $n = \frac{k_F^3}{3\pi^2}$ **[5]**

A6. Fermi energy **[2]**

A7**.** Electron-wave reflection from periodic lattice **[2]**

A8. Same periodicity as the Bravais lattice **[2]**

A9. N= number of atoms, a= interatomic spacing

$$2.\left(\frac{1}{2\pi}\right).Na.\left(\frac{2\pi}{a}\right) = 2N$$

Two spin orientations

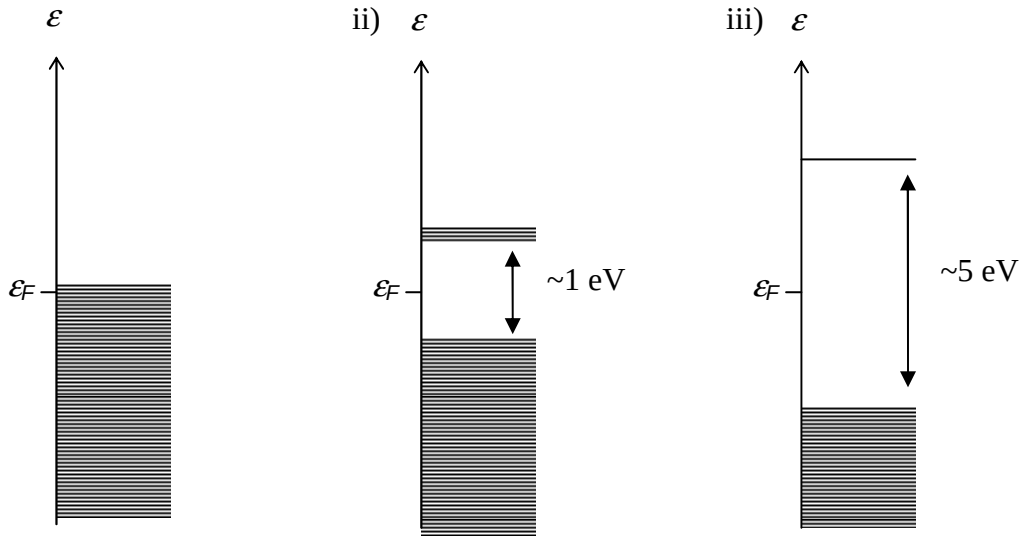
Real space length

k-space length of Brillouin zone

[5]

A10.

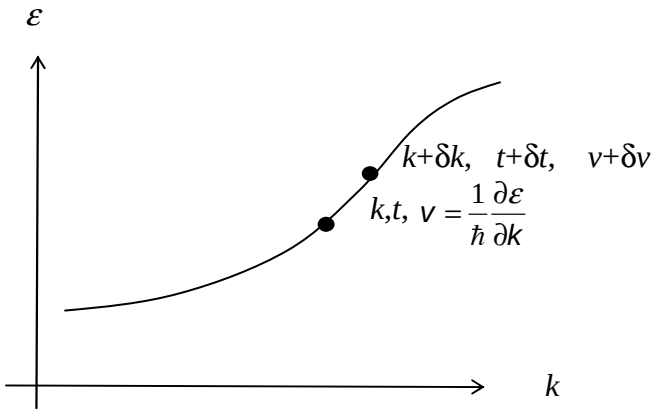
. i) ii) iii)



Metal **Semiconductor** **Insulator**

[2] **[2]** **[2]**

A11.



$$\delta v = \frac{1}{\hbar}\frac{\partial^2 \varepsilon}{\partial k^2}\delta k$$

$$a = \frac{\delta v}{\delta t} = \frac{1}{\hbar^2}\frac{\partial^2 \varepsilon}{\partial k^2} F = \frac{F}{m^*},$$

where m* is the effective mass, hence: $\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{\partial^2 \varepsilon}{\partial k^2}$ **[5]**

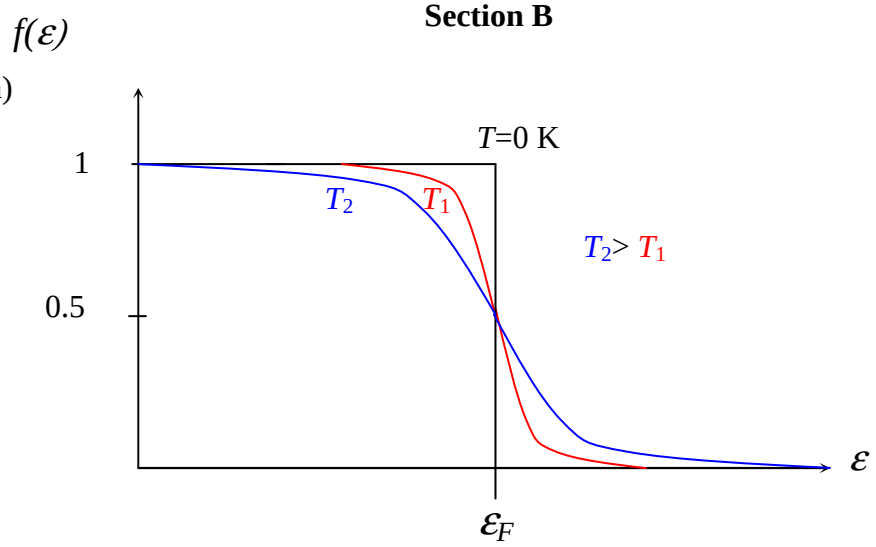
A12. Hole **[2]**

A13. i) Diffusion and drift currents **[2],** ii) diffusion current **[2]**

A14. Ultra-high vacuum chamber, molecular beam sources, lattice-matched substrate, growth-monitoring techniques. **[4]**

Section B

B1. a)



[2]

b) Add impurity atoms with +1 valence relative to host semiconductor; excess electron ionised by thermal energy and electron donated to conduction band. Add impurity atoms with -1 valence relative to host semiconductor; excess hole ionised by thermal energy and accepted by valence band. **[3]**

c) Assume CB occupancy <<1, $f(\varepsilon) \approx \frac{1}{e^{(\varepsilon-\varepsilon_F)/k_BT}} = e^{-\left(\frac{\varepsilon-\varepsilon_F}{k_BT}\right)}$

Probability of hole in VB, $1\text{-}\ f(\varepsilon) \approx 1 - \frac{1}{e^{(\varepsilon-\varepsilon_F)/k_BT}} = e^{-\left(\frac{\varepsilon_F-\varepsilon}{k_BT}\right)}$

Can use free electron concepts at bottom of conduction band and top of valence band, hence:

$$D_c(\varepsilon) = Cm^{*}_{c}{}^{\frac{3}{2}}(\varepsilon-\varepsilon_c)^{\frac{1}{2}}, \qquad D_v(\varepsilon) = Cm^{*}_{v}{}^{\frac{3}{2}}(\varepsilon_v-\varepsilon)^{\frac{1}{2}}$$

Where, m_c*=conduction band effective mass, m_v*=valence band effective mass, and C= constant

Number of electrons in CB,

$$n = \int_{\varepsilon_c}^{\infty} f.D_c\,d\varepsilon \approx Cm^{*}_{c}{}^{\frac{3}{2}} \int_{\varepsilon_c}^{\infty} (\varepsilon-\varepsilon_c)^{\frac{1}{2}}.e^{-\frac{(\varepsilon-\varepsilon_F)}{k_BT}}\,d\varepsilon \qquad \textbf{[5]}$$

substitute $y=(\varepsilon-\varepsilon_c)/k_BT$

$$n \approx C\,(m^*_c\, k_B T)^{\frac{3}{2}}.e^{-\frac{(\varepsilon_c-\varepsilon_F)}{k_B T}}\int_0^{\infty} y^{\frac{1}{2}}.e^{-y}dy$$

which gives,

$$n = 2\frac{(2\pi m^*_c\, k_B T)^{\frac{3}{2}}}{h^3}.e^{-\frac{(\varepsilon_c-\varepsilon_F)}{k_B T}} = N_c.e^{-\frac{(\varepsilon_c-\varepsilon_F)}{k_B T}} \qquad \textbf{[5]}$$

where N_c is the effective number density of *accessible* states at ε_c

Similarly the hole density, p is given by

$$p = 2\frac{(2\pi m^*_v\, k_B T)^{\frac{3}{2}}}{h^3}.e^{-\frac{(\varepsilon_F-\varepsilon_v)}{k_B T}} = N_v.e^{-\frac{(\varepsilon_F-\varepsilon_v)}{k_B T}} \qquad \textbf{[5]}$$

d) From c):

$$np \approx N_c.N_v.e^{-\frac{\varepsilon_g}{k_B T}} = W.T^3.e^{-\frac{\varepsilon_g}{k_B T}}$$

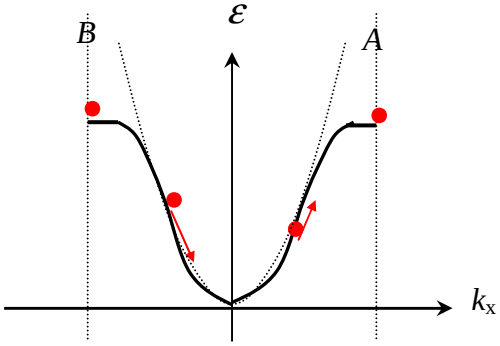
Significance: can calculate intrinsic carrier density, i.e. n=p

$$n_i = n = p = W^{\frac{1}{2}}T^{\frac{3}{2}}.e^{-\frac{\varepsilon_g}{2k_B T}} \qquad \textbf{[5]}$$

B2. a) Force, $\mathbf{F} = -e\mathbf{E}$

Equation of motion, $F = m\frac{dv}{dt} = \hbar\frac{dk}{dt}$, hence $\frac{dk}{dt} = -\frac{eE}{\hbar}$ **[5]**

b) A and B are equivalent states: real and k-space path of electron is a 1D oscillator.



[5]

c) Group velocity of a wave packet,

$$v_{\mathbf{k}} = \nabla_{\mathbf{k}}\omega = \frac{1}{\hbar}\frac{\partial \varepsilon_{\mathbf{k}}}{\partial \mathbf{k}} = \frac{1}{\hbar}\nabla_{\mathbf{k}}\varepsilon_{\mathbf{k}}$$

In 1D, $v_{\mathbf{k}} = \frac{1}{\hbar}\frac{\partial \varepsilon_{\mathbf{k}}}{\partial \mathbf{k}}$ **[5]**

d) Equation of motion, $F = m\frac{dv}{dt} = \hbar\frac{dk}{dt} \quad \Rightarrow \quad \delta k = -\frac{eE\tau}{\hbar}$ for each electron

If n electrons per unit volume and v_D= drift velocity

$$\mathbf{j} = n(-e)\boldsymbol{v}_D = n(-e)\left(-\frac{e\mathbf{E}\tau}{\hbar}\right)\left(\frac{\hbar}{m_e}\right) = \left(\frac{ne^2\tau}{m_e}\right)\mathbf{E} = \sigma\mathbf{E}$$

Electrical conductivity σ defined as $\mathbf{j} = \sigma.\mathbf{E}$

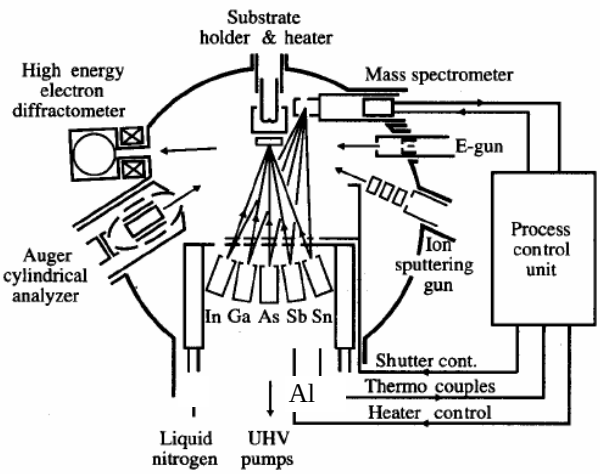
$$\Rightarrow \qquad \sigma = \frac{ne^2\tau}{m_e}$$ **[5]**

Mobility, the drift velocity per unit electric field, $\mu = |v_D|/E$, for electrons is defined as

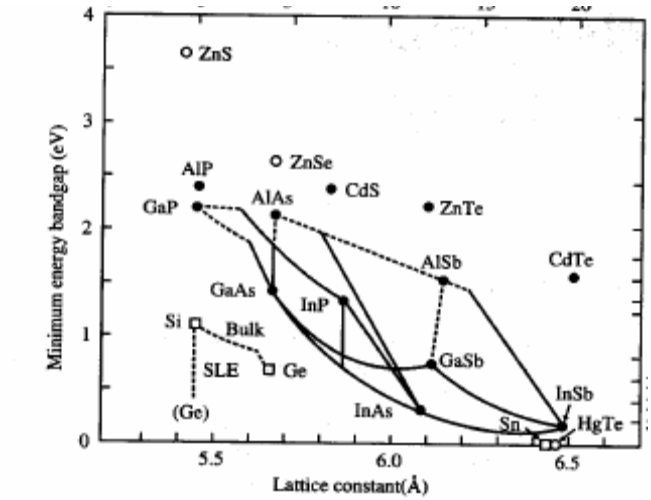
$\mu_e = \frac{e\tau_e}{m_e^*}$ and for holes $\mu_h = \frac{e\tau_h}{m_h^*}$

τ_e and τ_h are not necessarily the same. **[5]**

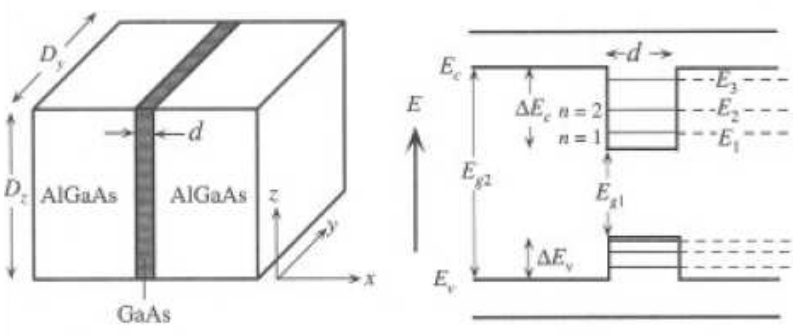
B3. a) MBE machine:



A quantum well comprises a layer of narrow band-gap semiconductor, such as GaAs, sandwiched between two wider band-gap semiconductors, such as AlGaAs (the complete structure is called a heterostructure). All semiconductors used to make the structure must be lattice-matched to maintain crystalline periodicity across the junction.



From the above diagram GaAs (narrow band-gap) and AlGaAs (wide band-gap) are lattice-matched. Since the band-gap energy changes at the interface there are discontinuities in the conduction band and valence band. These discontinuities, $\Delta\varepsilon_c$ and $\Delta\varepsilon_v$, depend on the semiconductor materials and their doping. In the case of a GaAs/AlGaAs heterostructure $\Delta\varepsilon_c$ is greater than $\Delta\varepsilon_v$, resulting in the band structure given below. Conduction electrons in the *x*-direction are confined due to the energy barrier $\Delta\varepsilon_c$. If the confinement length is small we can treat the electron as in a 1D potential well in the *x*-direction but as if it were free in the *yz* plane.



[10]

b) Current $I=veN/L$, where N = total number of electrons in 1D wire length L

Conductance, $G = I/V = veN/LV$

Drop in potential energy of electron going from one end to the other, $\Delta\varepsilon=eV$

$$\Rightarrow \quad G = \frac{ve^2N}{L\Delta\varepsilon}$$

N = 2 x number of quantum states in range $\Delta\varepsilon$

From Pauli principle

In 1D wire, only discrete values of electron wavelength λ are possible

$m\mathbf{v} = \hbar\mathbf{k} \Rightarrow$ ***mv = h/λ***

Electron velocity, $v = \dfrac{h}{\lambda m} \quad \Rightarrow \quad v_n = \dfrac{nh}{Lm}$

$\Rightarrow$ Number of electrons in velocity range Δv, $N = 2\dfrac{Lm\Delta v}{h}$ **[5]**

Also, kinetic energy of electron, $\varepsilon = \dfrac{mv^2}{2}$, $\Rightarrow \quad \Delta\varepsilon = mv\Delta v \quad \Rightarrow \; N = 2\dfrac{L\Delta\varepsilon}{vh}$

$\Rightarrow \quad G = \dfrac{2e^2}{h}$ Ohms^{-1}

$\Rightarrow$ Quantum of resistance=1/G=12.9kΩ **[5]**

c) Quantum conductance independent of all system parameters. **[5]**

B4. a) Metals have high free electron density $\Rightarrow$ positive ion cores are screened therefore free electron model is a good description of electron system. The low energy region of the nearly free electron dispersion relation approximates to the parabolic free electron dispersion relation therefore free electron concepts can be used to describe electrons in a semiconductor conduction band. **[5]**

b) (i) Using the 3D free electron Fermi gas model show that the expression for the Fermi energy in terms of the free electron density, n, is given by

$$\varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{\frac{2}{3}}$$

Fcc → 4 atom/unit cell → n=4/a^3= 8.5 x 10^{28} m^{-3}, put into above equation→ ε_F= 6.99 eV **[5]**

(ii) Number of electrons in Fermi disc, $N = 2\left(\pi k_F^2\right)\left(\frac{A}{4\pi^2}\right) = \frac{k_F^2}{2\pi}A$

k-space area

A=real space area

Electron density, $n = \frac{N}{A} = \frac{k_F^2}{2\pi} \Rightarrow \boxed{\varepsilon_F = \frac{\hbar^2 k_F^{\ 2}}{2m} = \frac{\hbar^2}{2m}(2\pi n) = 0.7 \text{ meV}}$ **[5]**

(iii) Electrons density = $1/(0.8 \times 10^{-9}) = 1.25 \times 10^9 \text{ m}^{-1}$

Number of electrons in Fermi length = $2\left(2k_F\right)\left(\frac{L}{2\pi}\right) = \frac{2k_F}{\pi}L$

k-space length L=real space length

Electron density, $n = \frac{N}{L} = \frac{2k_F}{\pi} \Rightarrow \boxed{\varepsilon_F = \frac{\hbar^2 k_F^{\ 2}}{2m} = \frac{\hbar^2}{2m}\left(\frac{\pi n}{2}\right)^2 = 146 \text{ meV}}$ **[5]**

(c) Use formula from part (b) to find ε_F =6.96 eV
Thermal energy is $k_B T$
Room temperature≈300 K
$\Rightarrow$ $k_B T = 0.025$ eV
$\varepsilon_F / k_B T \approx 280 \Rightarrow$ gas is highly degenerate **[5]**