Queen Mary
University of London

BSc/MSci Examination

PHY-550 Solid State Physics

Time allowed: 2 hours 30 minutes

Date: 19th May 2010

Time: 10:00

Instructions: **Answer ALL questions in section A. Answer ONLY TWO questions from section B. Section A carries 50 marks, each question in section B carries 25 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question. Course work comprises 20% of the final mark**

You may wish to use the following information:

Electron charge	e	-1.6×10^{-19}	C
Electron mass	m_e	9.11×10^{-31}	kg
Boltzmann's constant	k_B	1.38×10^{-23}	JK^{-1}
Planck's constant	h	6.62×10^{-34}	Js
	$\hbar = h/2\pi$	1.05×10^{-34}	Js
Permitivity of free space	ε_0	8.85×10^{-12}	Fm^{-1}

Numeric calculators are permitted in this examination. Please state on your answer book the name and type of machine used. Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Important note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from the college. Please check now to ensure you do not have any notes in your possession. If you have any then please raise your hand and give them to an invigilator immediately. Exam papers cannot be removed from the exam room

You are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Examiners: Dr. M. Baxendale, Dr T.J.S. Dennis

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SECTION A. Attempt answers to all questions

A1.

a) Sketch the unit cell and state the number of lattice points per unit cell for the face-centred cubic Bravais lattice. **[2 marks]**

b) Label the primitive lattice vectors on the same diagram. **[2 marks]**

A2. The structure of crystalline gallium arsenide (GaAs) can be described as a face-centred cubic lattice with a basis of Ga at (0,0,0) and As at (**a**/4)+(**b**/4)+(**c**/4), where **a**,**b**, and **c** are unit lattice vectors. Sketch the GaAs structure. **[4 marks]**

A3. Sketch the following lattice planes for a cubic lattice: (011), (102), (211). **[3 marks]**

A4. What is the main assumption of the free electron Fermi gas model of electron motion in solids?

[2 marks]

A5. The three dimensional k-space representation of the ground state of a free electron Fermi gas is spherical with radius equal to the Fermi wavevector, k_F. Each allowed state occupies $8\pi^3/V$ k-space volume, where V is the real space volume. Derive an expression for the electron density, n.

[5 marks]

A6. Name the electron energy at which the probability of occupancy of an electron state is 0.5 for a system in thermal equilibrium at any temperature? **[2 marks]**

A7. In an infinite one dimensional lattice what physical process dominates electron motion as the wavevector approaches the Brillouin zone boundary at $\pm\pi/a$, where a is the lattice spacing?

[2 marks]

A8. According to Bloch's theorem what is the periodicity of electron wavefunctions in the nearly free electron model? **[2 marks]**

A9. In one dimension there are $1/2\pi$ states per unit length of real space per unit length of k-space. Show that there are $2N$ electron states per band, where N is the number of atoms in a lattice. **[5 marks]**

A10. Using simple band diagrams and by indicating the position of the Fermi energy explain what is meant by the terms:

a) metal, **[2 marks]**

b) semiconductor, **[2 marks]**

c) insulator. **[2 marks]**

A11. By analogy with Newton's second law prove that the effective mass of an electron in motion in a solid is given by:

$$\frac{1}{m*} = \frac{1}{\hbar^2}\frac{\partial^2 \varepsilon}{\partial k^2},$$

where ε is energy and k is wavevector. **[5 marks]**

A12. What will be the majority carrier in silicon doped with 10^{15} cm^{-3} atoms of valence three?

[2 marks]

A13.

a) What are the sources of current flow in unbiased p-n junction that result in zero net current?

[2 marks]

b) Which source or sources of current dominate when the p-n junction is forward biased? **[2 marks]**

A14. Name four essential features of a molecular beam epitaxy system for the growth of ordered semiconductor layers. **[4 marks]**

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SECTION B. Attempt two of the four questions in this section

B1.

a) The Fermi-Dirac distribution as a function of energy ε, for a system at temperature T is given by,

$$f(\varepsilon) = \frac{1}{e^{(\varepsilon - \varepsilon_F)/k_B T} + 1},$$

where ε_F is the Fermi energy of the system.

Sketch the distribution for $T = 0$ and $T >> 0$. **[2marks]**

b) How can doping be used to control the properties of a semiconductor? **[3 marks]**

c) In a three dimensional free electron system the number of states per unit energy, per unit volume of real space is given by:

$$D(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{\frac{3}{2}} \varepsilon^{\frac{1}{2}},$$

where ε is the energy and m_e is the rest mass of the electron.

Derive expressions for hole and electron densities, p and n, in a semiconductor in terms of integrals over $D_V(\varepsilon)$ and $D_C(\varepsilon)$, the densities of states per unit energy per unit volume of real space for holes and electrons, respectively. **[15 marks]**

(Standard integral, $\int_0^\infty y^{\frac{1}{2}} e^{-y} dy = \frac{1}{2}\sqrt{\pi}$)

d) Show that the product of p and n is independent of ε_F and comment on the significance of the result. **[5 marks]**

B2.

a) Show that an electron of charge e and mass m changes its **k** vector in response to an electric field **E** at a rate $d\mathbf{k}/dt=-e\mathbf{E}/\hbar$. **[5 marks]**

b) Describe the concept of the Bloch oscillator and give a possible reason why it has not yet been detected experimentally. **[5 marks]**

c) The Bloch states $\Psi_k(r)$ describing the conduction and valence bands of a semiconductor have energies $\varepsilon(k)$.

Show the velocity v of an electron in a Bloch state in one dimension is $(1/\nabla)\ \ \partial\varepsilon/\partial k$. **[5 marks]**

d) Derive expressions for electron or hole mobility in a semiconductor in terms of the effective mass and scattering time. **[10 marks]**

B3.

a) Explain how a quantum well can be made from semiconductor layers. **[10 marks]**

b) A small voltage is applied to two macroscopic contacts that are connected by an ideal quantum wire comprising a few tens of atoms. Show that the electrical conductance G measured across the contacts is given by

$$G = \frac{2e^2}{h}$$

where e is the electronic charge and h is Plank's constant. Hence calculate the quantum of resistance. **[10 marks]**

c) Explain qualitatively how the expression differs from that for electrical conductivity for an infinite one-dimensional array of atoms. **[5 marks]**

B4.

a) Explain why the free electron model of electron motion in periodic structures closely describes the electronic properties of metals and low energy electrons in the conduction band of semiconductors. **[5 marks]**

b) Using the free electron model calculate the Fermi energy for the following:

(i) Copper, which has face centred cubic structure with lattice spacing a=0.361 nm and for which one atom contributes one free electron. **[5 marks]**

(ii) A two dimensional solid containing 3×10^{15} m^{-2} free electrons. **[5 marks]**

(iii) A one dimensional conductor whose unit cells each contribute one free electron and are 0.8 nm long. **[5 marks]**

c) Calculate the value of the ratio of the Fermi energy in Cu to the thermal energy at room temperature using the electron density n=8×10^{28} m^{-3} and confirm that the electron gas is highly degenerate. **[5 marks]**

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