Queen Mary
University of London

BSc/MSci EXAMINATION

PHY-410 The Interstellar Medium

Time Allowed: 2 hours 15 minutes

Date: 18 May 2005

Time: 14.30-16.45

Answer ALL questions in Section A (total of 40 marks) and any TWO of the four questions in Section B (each 30 marks).

An indicative marking-scheme is shown in square brackets [ ] after each part of a question.

Numeric calculators may be used.

Formulae: You may use any of the following expressions. Throughout the paper symbols have the usual meaning.

A) $I_\nu = I_\nu(0)e^{-\tau_\nu} + S_\nu[1 - e^{-\tau_\nu}]$

B) $\tau_\nu = \dfrac{c^2}{8\pi\nu^2} A_{ul}(e^{h\nu/kT_{ex}^{ul}} - 1)N_u\phi_\nu$

C) $B_\nu(T) = \dfrac{2h\nu^3}{c^2}\dfrac{1}{e^{h\nu/kT} - 1}$

D) $e^x \approx 1 + x$ for $|x| << 1$

E) $m_{obs} = M + A + (5\log D - 5)$ where D is in pc

Constants:

1 parsec = 1 pc = 3.09×10^{16} m
Atomic mass of hydrogen $m_H = 1.66\times10^{-27}$ kg
$\log_{10}e = 0.434$

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Section A

1. The number density of particles in the interstellar medium is much smaller than that typically found in a laboratory 'vacuum tube'. State why the interstellar medium nevertheless behaves like a gas rather than the laboratory vacuum. **[3 marks]**

2. Write down an equation defining the excitation temperature between levels u and l. **[3 marks]**

3. Explain what is meant by column density and give the equation relating it to number density. **[3 marks]**

4. Given $\kappa_\nu = \frac{c^2}{8\pi\nu^2} A_{ul} \left(\frac{g_u n_l}{g_l n_u} - 1 \right) n_u \phi_\nu$ and $j_\nu = \frac{h\nu}{4\pi} A_{ul} n_u \phi_\nu$, where κ_ν and j_ν are the volume opacity and volume emissivity respectively, show that the source function S_ν equals the Planck function at a temperature equal to the excitation temperature of the transition. **[5 marks]**

5. Write down an equation *defining* the optical depth τ_ν at frequency ν. Hence obtain an expression for the optical depth of a *uniform* molecular cloud of total thickness s m, in terms of the column density and other quantities, using any of the expressions quoted or obtained in questions 3 & 4. **[4 marks]**

6. What is meant by *degeneracy* of an energy level? What is the degeneracy of the rotational level of a diatomic molecule with angular momentum quantum number J? **[4 marks]**

7. What is responsible for the energy difference between the two states producing the 21cm line of atomic hydrogen? **[2 marks]**

8. Explain why ¾ of HI atoms in the interstellar medium are in the upper spin state, justifying explicitly each step in your argument. **[5 marks]**

9. For a uniform cloud with no background source obtain expressions for the specific intensity (a) in the optically thin limit and (b) in the optically thick limit. Hence explain how the excitation temperature can be estimated from observations of a cloud of large optical depth. **[6 marks]**

10. What do the Einstein coefficients A_{ul} & B_{ul} describe respectively, and what are the units of each? **[5 marks]**

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SECTION B.

Results obtained in Section A may be quoted without proof, but extra credit will not be given for repetition of material already answered in Section A.

B.1

Explain briefly why Thermodynamic equilibrium is not generally achieved in the interstellar medium, explaining physically what are meant by the terms excitation, radiation and kinetic temperatures (T_{rad}, T_{ex} & T_{kin}) indicating why they are useful but distinct.

[9 Marks]

Write down, defining the symbols used, the Equation of Statistical Equilibrium for a two level atom (or molecule) whose relative energy level populations n_u and n_l are determined by three processes:

a) collisions with particles of density n and kinetic temperature T_{kin};
b) interaction with an ambient radiation field energy density U_ν at frequency ν;
c) spontaneous emission of radiation. **[5 marks]**

Hence find an expression for n_u/n_l in terms of U_ν, and the ratios of B_{ul}, B_{lu}, C_{ul}, C_{lu} to A_{ul}.
Define the critical density n_{crit}, providing a physical motivation.
Hence show that if the energy density of the radiation field at frequency ν ($U_\nu = 4\pi I_\nu / c$) can be described in terms of blackbody radiation at a radiation temperature T_{rad} and a dilution factor W, then

$$e^{-h\nu/kT_{exc}} = \frac{\{[n/n_{crit}]e^{-h\nu/kT_{kin}} + W/[e^{h\nu/kT_{rad}} - 1]\}}{\{[n/n_{crit}] + W/[e^{h\nu/kT_{rad}} - 1] + 1\}}$$

where $h\nu$ is energy difference between levels u and l. **[10 marks]**

In your derivation you may assume without proof that:
(a) the dependence of collision rate coefficients on the number density of collision partners is

$$C_{ul} = n\gamma_{ul}$$

(b) $$\frac{C_{lu}}{C_{ul}} = \frac{g_u}{g_l} e^{-h\nu/kT_{kin}} \text{ and the Einstein relations}: B_{lu} = \frac{g_u}{g_l} B_{ul} \text{ and } \frac{B_{ul}}{A_{ul}} = \frac{c^3}{8\pi h\nu^3}$$

By considering the relative importance of collisional and radiative processes show under what conditions the excitation temperature approaches respectively the kinetic and radiation temperature. **[6 marks]**

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B.2

Explain what 'HII regions' are. What is the evidence that they occur in regions of recent star formation? **[5 marks]**

Describe the basic processes of photoionization and recombination in a typical HII region. Show, stating clearly the assumptions made, that the radius of an ionized region of number density n_o surrounding a young star which emits S_* ionizing photons/sec is:

$$R_S = \left(\frac{3S_*}{4\pi n_0^2 \alpha_B} \right)^{\frac{1}{3}}$$

where $\alpha_B = 2 \times 10^{-19}$ m^3 s^{-1} is the recombination coefficient into states with $n = 2,3...$
Explain why the recombination coefficient here does not include recombination to the ground state $n = 1$.
Explain why the edges of HII regions are sharply defined. **[13 marks]**

Describe the process of free-free radiation. The optical depth for free-free radiation at frequency ν, of an ionized hydrogen cloud of thickness L can be approximated by: $\tau_\nu \approx \text{const}\ \nu^{-2.1}\ T_e^{-1.35}\ n_e^2\ L$ where T_e and n_e are the electron temperature and electron density respectively.

Using this information with the solution to the equation of radiative transfer (given on the title page) derive expressions which show how the intensity of the thermal radio continuum emission from an ionized hydrogen cloud varies with frequency at low and high radio frequencies. State any assumptions clearly, including justification of the use of the Rayleigh-Jeans approximation. **[12 marks]**

B.3

Sketch the shape of the interstellar extinction curve over the full observed wavelength range, labelling the axes, indicating clearly the IR, visible and UV regions and noting any significant features and the type of dust grains responsible for these features. What is the approximate dependence of the extinction on wavelength for large and small wavelengths? **[10 Mark]**

Given that the above curve is a plot of $E_{\lambda-V}/E_{B-V} = (A_\lambda - A_V)/(A_B - A_V)$, where A_λ is the extinction at wavelength λ, explain how the relation A_V=3.1 E_{B-V} is found from such a curve. **[5 Marks]**

Starlight in our part of the galaxy typically suffers 1.2 magnitude of visual extinction, A_V, by dust for each kpc travelled. Show that the corresponding optical depth of the dust per kpc is

$$\tau = 1.2/2.5\log_{10} e$$ **[5 marks]**

The dust optical depth is given by $\tau = Q_{ext}\, \pi\, a^2\, N_d\, L$ where the extinction efficiency is typically $Q_{ext} \sim 0.9$, the average radius of the grains is $a \sim 0.3$ μm, N_d is the average dust particle number density, and L is the total path length of the radiation through the dust.

Calculate the number density and hence the mass density of dust grains in the interstellar medium, assuming that the average solid density of an interstellar dust grain is $\rho_g = 3000$ kg m^{-3}. Find how this compares with the mass density of gas, given that the number density of hydrogen in the ISM is n_H~2×10^6 m^{-3}. **[10 marks]**

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B.4

Star formation is observed to occur in the Galaxy at a rate some 50 times less than expected. What possibilities may account for the observed star formation rate?

[3 marks]

The virial theorem states that $2U = -\Omega$ where $2U = 3\int PdV$. Explain in general terms how this result is obtained, defining the symbols and giving physical assumptions involved.

Show that in the simple case of a spherical cloud with constant density, ρ,

$$\Omega = -\frac{3}{5}\frac{GM_{cloud}^2}{R_{cloud}}$$ **[9 marks]**

Use the virial theorem to state a general condition for an isolated cloud to collapse.

Hence, given that $2U \approx 3M_{cloud}c_s^2$, where the sound speed is $c_s = \sqrt{\frac{kT}{\mu m_H}}$, show that for the constant density cloud this condition can be expressed in terms of the sound crossing time as

$$t_{cross} > \left(\frac{15}{4\pi G\rho}\right)^{\frac{1}{2}}$$ **[5 marks]**

By considering the free-fall collapse of this constant density cloud, *and assuming* for simplicity that the *acceleration remains constant* throughout the collapse, show that

$$t_{ff} \approx \left(\frac{3}{2\pi G\rho}\right)^{\frac{1}{2}}$$

Comment on why this is only slightly longer than the more exact result,

$$t_{ff} = \left(\frac{3\pi}{32G\rho}\right)^{\frac{1}{2}},$$ **[9 marks]**

which takes account of the increasing acceleration as the cloud collapse proceeds.

Hence express the cloud collapse condition in terms of t_{cross} and t_{ff} and use your result to comment on the interpretation of this condition in physical terms. **[4 marks]**

End of Examination Paper **Dr P.G. Williams**

