Queen Mary
University of London

BSc/MSci EXAMINATION

PHY-410 The Interstellar Medium

Time Allowed: 2 hours 15 minutes

Date: 9 May 2007

Time: 14.30-16.45

Answer ALL questions in Section A (total of 40 marks) and any TWO of the four questions in Section B (each 30 marks). An indicative marking-scheme is shown in square brackets [ ] after each part of a question.

COMPLETE ALL ROUGH WORKINGS IN THE ANSWER BOOK AND CROSS THROUGH ANY WORK WHICH IS NOT TO BE ASSESSED.
NUMERIC CALCULATORS ARE PERMITTED IN THIS EXAMINATION.

Formulae: You may use any of the following expressions. Here and throughout the paper symbols have the usual meaning.

$$U_\nu = \frac{4\pi}{c} I_\nu \quad \text{.......... (1)}$$

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu[1 - e^{-\tau_\nu}] \quad \text{.......... (2)}$$

$$\tau_\nu = \frac{c^2}{8\pi\nu^2} A_{ul}(e^{h\nu/kT_{ex}^{ul}} - 1)N_u\phi_\nu \quad \text{.......... (3)}$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}, \quad \text{.......... (4)}$$

$$e^x \approx 1 + x \text{ for } |x| << 1 \quad \text{.......... (5)}$$

$$m_{obs} = M + A + (5\log D - 5) \text{ where } D \text{ is in pc} \quad \text{.......... (6)}$$

Constants:
1 parsec = 1 pc = 3.09×10^{16} m
Atomic mass of hydrogen $m_H = 1.66\times10^{-27}$ kg

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR

Section A Answer ALL questions in this Section.

A1. The number density of particles in the interstellar medium is much smaller than that typically found in a laboratory 'vacuum tube'. State why the interstellar medium nevertheless behaves like a gas rather than the laboratory vacuum. **[3]**

A2. Explain briefly why thermodynamic equilibrium is not generally achieved in the interstellar medium, explaining physically what are meant by the terms excitation, radiation and kinetic temperatures (T_{rad}, T_{ex} & T_{kin}). What is the range of temperature values found in typical molecular clouds? **[9]**

A3. Name the two most abundant molecules in the interstellar medium. Why is the most abundant of these difficult to detect, but the other easy? **[4]**

A4. What is meant by *degeneracy* of an energy level? What is the degeneracy of the rotational level of a diatomic molecule with angular momentum quantum number J? **[3]**

A5. What is responsible for the energy difference between the two states producing the 21cm line of atomic hydrogen? **[2]**

A6. Show that ¾ of HI atoms in the interstellar medium are in the upper spin state, justifying explicitly each step in your argument. **[6]**

A7. For a uniform cloud with no background source obtain expressions for the specific intensity (a) in the optically thin limit and (b) in the optically thick limit. Hence explain how the excitation temperature can be estimated from observations of a cloud of large optical depth. **[4]**

A8. Write down, defining the symbols used, the Equation of Statistical Equilibrium for a two level atom (or molecule) whose energy level populations n_u and n_l are determined by three processes: a) collisions with rate coefficient C_{ul} per atom; b) interaction with an ambient radiation field of energy density U_ν at frequency ν; c) spontaneous emission of radiation. **[4]**

A9. The equation of statistical equilibrium, taking account of both collisions and interaction with a black body radiation field at a radiation temperature T_{rad} diluted by a factor W, leads to the condition

$$e^{-h\nu/kT_{exc}} = \frac{\{[n/n_{crit}]e^{-h\nu/kT_{kin}} + W/[e^{h\nu/kT_{rad}} - 1]\}}{\{[n/n_{crit}] + W/[e^{h\nu/kT_{rad}} - 1] + 1\}}, \text{ where the critical density is } n_{crit} = \frac{A_{ul}}{\gamma_{ul}}$$

Hence find the excitation temperature in the following two limits: (a) the radiation field is strong and dominates, (b) collisions dominate. **[5]**

Please turn to the next page.

Section B Answer TWO questions in this Section.

B.1

(a) Given the equation for radiative transfer

$$dI_\nu = -\kappa_\nu I_\nu ds + j_\nu ds\ ,$$

explain the physical significance of the individual symbols ds, I_ν, κ_ν, j_ν. Explain the physical content of the equation by stating what dI_ν and each of the two terms on the right hand side represent. **[7]**

(b) Define the optical depth τ_ν and hence, by multiplying the radiative transfer equation by an integrating factor $exp(\tau_\nu)$, derive its formal solution (name and define the quantity S_ν):

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu d\tau_\nu$$

[5]

Hence obtain an expression for $I_\nu(\tau_\nu)$ in the case of a *uniform* cloud of optical depth τ_ν. **[3]**

(c) κ_ν is given by

$$\kappa_\nu I_\nu = (n_l B_{lu} - n_u B_{ul}) U_\nu \frac{h\nu}{4\pi}\phi_\nu$$

Explain the physical origin of this expression and in particular the reason why the quantities,
$n_l B_{lu} U_\nu$, $n_u B_{ul} U_\nu$ occur. Why is there a minus sign for the $n_u B_{ul}$ term. **[4]**

(d) j_ν is given by

$$j_\nu = n_u A_{ul} \frac{h\nu}{4\pi}\phi_\nu$$

Explain the physical origin of this expression and in particular the reason why the quantity $n_u A_{ul}$ occurs. **[2]**

(e) Use the above expression for κ_ν and the Einstein relations

$$B_{lu} = \frac{g_u}{g_l} B_{ul} \text{ and } B_{ul} = \frac{c^3}{8\pi h \nu^3} A_{ul}$$

to obtain the expression **[4]**

$$\kappa_\nu = \frac{c^2}{8\pi\nu^2} A_{ul}\left(\frac{g_u n_l}{g_l n_u} - 1\right) n_u \phi_\nu$$

Using the above expressions for j_ν, κ_ν and the definition of the excitation temperature, T_{ex}, show that S_ν equals the Planck function at a temperature equal to the excitation temperature of the transition. **[5]**

Please turn to the next page.

B.2

(a) Explain what HII regions are. What is the evidence that they occur in regions of recent star formation? **[4]**

(b) Describe the basic processes of photoionization and recombination in a typical HII region. Show, giving a detailed argument and stating clearly the assumptions made, that the radius of an ionized region of number density n_o surrounding a young star which emits S_* ionizing photons/sec is: **[7]**

$$R_S = \left(\frac{3S_*}{4\pi n_0^2 \alpha_B} \right)^{\frac{1}{3}}$$

where $\alpha_B = 2 \times 10^{-19}$ m^3 s^{-1} is the recombination coefficient into states with $n = 2,3\ldots$ Explain why the recombination coefficient here does not include recombination to the ground state $n = 1$. **[3]**

(c) Explain why the edges of HII regions are sharply defined. **[2]**

(d) Describe the process of free-free radiation. The optical depth for free-free radiation at frequency ν, of an ionized hydrogen cloud of thickness L can be approximated by:

$$\tau_\nu \approx A\,\nu^{-2.1} T_e^{-1.35} n_e^2 L$$

where T_e and n_e are the electron temperature and electron density respectively and A is a known constant.
Using this information with the solution to the equation of radiative transfer (eq.(2) on the title page) derive expressions which show how the intensity of the thermal radio continuum emission from an ionized hydrogen cloud varies with frequency at low and high radio frequencies. State any assumptions clearly, including justification of the use of the Rayleigh-Jeans approximation. **[9]**

(e) Use your results to sketch the predicted radio spectrum (in a $\log I_\nu$ vs $\log \nu$ plot) from an HII region. Use this sketch to explain how a measurement of the spectrum together with an estimate of L and T_e (and T_{rad}) yields an estimate for the electron density n_e. **[5]**

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B.3

(a) Sketch the shape of the interstellar extinction curve over the full observed wavelength range, labelling the axes, indicating clearly the IR, visible and UV regions and noting any significant features and the type of dust grains responsible for these features. What is the approximate dependence of the extinction on wavelength for large and small wavelengths? **[10]**

(b) Given that the above curve is a plot of $E_{\lambda\text{-}V}/E_{B\text{-}V} = (A_\lambda - A_V)/(A_B - A_V)$, where A_λ is the extinction at wavelength λ, explain how the relation A_V=3.1 $E_{B\text{-}V}$ is found from such a curve. **[5]**

(c) Starlight in our part of the galaxy typically suffers 1.2 magnitude of visual extinction, A_V, by dust for each kpc travelled. Show that the corresponding optical depth of the dust per kpc is

$$\tau = 1.2/2.5\log_{10} e$$

[5]

(d) The dust optical depth is given by $\tau = Q_{ext}\,\pi\, a^2\, N_d\, L$ where the extinction efficiency is typically $Q_{ext} \sim 0.9$, the average radius of the grains is $a \sim 0.3$ μm, N_d is the average dust particle number density, and L is the total path length of the radiation through the dust.
Calculate the number density and hence the mass density of dust grains in the interstellar medium, assuming that the average density of an interstellar dust grain is typically ρ_g=3000 kg m^{-3}. Find how this compares with the mass density of gas, given that the number density of hydrogen in the ISM is $n_H \sim 2\times10^6$ m^{-3}. **[10]**

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B.4

(a) Star formation is observed to occur in the Galaxy at a rate some 50 times less than expected. What possibilities may account for the observed star formation rate? **[3]**

(b) The virial theorem states that $2U = -\Omega$ where $2U = 3\int PdV$. Explain in general terms how this result is obtained, defining the symbols and giving physical assumptions involved.

Show that in the simple case of a spherical cloud with constant density, ρ, **[9]**

$$\Omega = -\frac{3}{5}\frac{GM_{cloud}^2}{R_{cloud}}$$

(c) Use the virial theorem to state a general condition for an isolated cloud to collapse.

Hence, given that $2U \approx 3M_{cloud}c_s^2$, where the sound speed is $c_s = \sqrt{\frac{kT}{\mu m_H}}$, show that for the constant density cloud this condition can be expressed in terms of the sound crossing time as the simple case of a spherical cloud with constant density, ρ, **[5]**

$$t_{cross} > \left(\frac{15}{4\pi G\rho}\right)^{\frac{1}{2}}$$

(d) By considering the free-fall collapse of this constant density cloud, *and assuming* for simplicity that the *acceleration remains constant* throughout the collapse, show that

$$t_{ff} \approx \left(\frac{3}{2\pi G\rho}\right)^{\frac{1}{2}}$$

Comment on why this is only slightly longer than the more exact result,

$$t_{ff} = \left(\frac{3\pi}{32G\rho}\right)^{\frac{1}{2}},$$

which takes account of the increasing acceleration as the cloud collapse proceeds. **[9]**

(e) Hence express the cloud collapse condition in terms of t_{cross} and t_{ff} and use your result to comment on the interpretation of this condition in physical terms. **[4]**

End of Examination Paper **Dr P.G. Williams**