Queen Mary
University of London

BSc/MSci EXAMINATION

PHY-302 Nuclear Physics And Astrophysics

Time Allowed: 2 hours 15 minutes

Date: 16th May 2007

Time: 10:00

Answer ALL questions in section A. Answer ONLY TWO questions from section B. Section A carries 40 marks, each question in section B carries 30 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question.

COMPLETE ALL ROUGH WORKINGS IN THE ANSWER BOOK AND CROSS THROUGH ANY WORK WHICH IS NOT TO BE ASSESSED.

NUMERIC CALCULATORS ARE PERMITTED IN THIS EXAMINATION.

Data

Electronic charge	**e**	**1.602×10^{-19} C**	
Electron mass	**m_e**	**0.511003 MeV/c^2**	**5.485803×10^{-4} u**
Proton mass	**m_p**	**938.280 MeV/c^2**	**1.00727647 u**
Neutron mass	**m_n**	**939.573 MeV/c^2**	**1.00866501 u**
Atomic mass unit	**u**	**931.502 MeV/c^2**	
Avogadro constant	**N_a**	**6.022045×10^{23} g mol^{-1}**	
electron volt	**eV**	**1.602189×10^{-19} J**	
Boltzmann constant	**k**	**8.6174×10^{-11} MeV/K**	

Mass	**$^{236}_{92}U$**	**236.045568 u**
Mass	**$^{235}_{92}U$**	**235.043924 u**

$e^2/(4\pi\varepsilon_0) = 1.44$ MeV fm

If needed assume the ordering of nuclear shells to be

$1s_{1/2}$; $1p_{3/2}$; $1p_{1/2}$; $1d_{5/2}$; $1d_{3/2}$; $2s_{1/2}$; $1f_{7/2}$; $1f_{5/2}$; $2p_{3/2}$; $2p_{1/2}$; $1g_{9/2}$; $1g_{7/2}$; $2d_{5/2}$; $2d_{3/2}$; $1h_{11/2}$; $3s_{1/2}$;$1h_{9/2}$;

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

SECTION A: Answer all questions in this section

A1. Describe in words what is meant by the thermal reproduction factor k_∞. Give the equation defining k_∞ and describe each of the four terms. [5]

A2. Give an approximate value in metres for the nuclear radius of ^{36}Cl showing how you arrived at your answer. Estimate the atomic radius of ^{36}Cl ? [4]

A3. Describe in words what is meant by the cross section of a nuclear process. [2]

A4. Write down the three reaction equations for β^+, β^-, and ε (electron capture) processes. [3]

A5. Describe briefly the <u>difference</u> between the pp and CNO cycles of hydrogen burning in stars. Reaction equations and Q values are <u>not</u> required. Does the CNO cycle dominate in hotter or colder stars? Explain why. [4]

A6. Give the formula used to calculate the mass of a nucleus M(Z,A) given the binding energy B. Calculate the mass of Mg which has Z=12 and A=26 if the binding energy is 218 MeV/c^2. [4]

A7. Explain why alpha particles have a much shorter range in air than beta particles of similar energy. [2]

A8. Define the Q of a nuclear reaction in terms of nuclear masses. By considering the decay reaction $A \rightarrow B + C$ derive the Q value in terms of initial and final kinetic energies. What interpretation should be given to a reaction which has a negative Q value? [4]

A9. Describe briefly what is meant by the r and s processes of nucleosynthesis. What are their main distinguishing features? What conditions are required for these two processes? [4]

A10. Considering a hot gas of $^{10}_{5}$B nuclei, calculate classically the temperature required to overcome the Coulomb barrier and the energy release in the fusion reaction. Why are stellar fusion reactions observed to occur at much lower temperatures? [4]

A11. Deduce from the shell model the spin and parity assignments for the ground states of the following nuclei:

a) $^{11}_{5}$B

b) $^{17}_{9}$F [4]

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SECTION B: Answer two questions from this section

B1)

a) For a sample of N nuclei present at time t, the number of nuclei dN decaying in a time dt is given by a differential equation characterising the decay constant λ. Write down this equation and its general solution for $N(t)$. **[3]**

b) Give equations for the mean lifetime τ, and the half life $t_{½}$ in terms of the decay constant λ. Describe in words the meaning of half life. Sketch a graph of N vs t for a sample of 10^{24} nuclei which undergo radioactive decay with a decay constant of 0.005 days^{-1}. Show on the graph the number of nuclei left after a time $t = \tau$ and $t = t_{½}$. **[5]**

c) $^{212}_{83}$Bi undergoes α decay to Titanium (Ti) and β^- decay to Polonium (Po). Write down the equations for both of these reactions giving the A and Z values. If the half life of α decay is 14.71 s and the total decay rate is 0.536 s^{-1} what is the half life of the β^- decay mode? **[5]**

d) $^{210}_{83}$Bi has a mean lifetime of 7.2 days and decays via β-particle emission to ^{210}Po which in turn decays via α-particle emission to ^{206}Pb with a mean lifetime 200 days. If a source initially contains pure $^{210}_{83}$Bi, after how long will the rate of α-particle emission reach a maximum? Derive any equations you use showing your working. You may assume a trial solution of the form $Ae^{-\lambda_1 t} + Be^{-\lambda_2 t}$. **[10]**

e) If a sample of 1 gram of ^{210}Po exists at the present time, estimate the number of atoms of Po at the present time and after 100 days. What is the mass of the Polonium sample after 100 days? **[5]**

f) Given the relatively short half life of ^{210}Po state (without calculation) whether or not you would expect the Q of it's alpha decay to be large or small giving your reason. **[2]**

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B2)

a) Explain, in terms of nuclear binding energy, why fusion processes are generally energetically more favourable for light nuclei whereas fission reactions are only important for heavy nuclei. **[2]**

b) $^{235}_{92}U$ is an α emitter with a half life of 10^9y. It is also able to undergo spontaneous fission with a half life of 10^{16}y. What is meant by the term spontaneous fission? **[2]**

c) Calculate the Coulomb potential energy for $^{235}_{92}U$ briefly existing in the form of:

- **i.** An alpha particle and an isotope of $_{90}Th$.
- **ii.** A $^{117}_{46}Pd$ nucleus and a $^{118}_{46}Pd$ nucleus.

By sketching the shape of the nuclear potential as a function of radial distance r, explain which mechanism is responsible for spontaneous fission and why this is suppressed for large mass fragments. **[6]**

d) Fission may be induced by the collision of a neutron with the Uranium nucleus. $^{235}_{92}U$ has a large cross section for absorption of thermal neutrons leaving the new uranium isotope in an excited state with a mass of 219883.5 MeV/c^2. Calculate the excitation energy. Explain what is meant by the activation energy for the fission of $^{236}_{92}U$. If the activation energy is found to be 6.2 MeV what is the minimum kinetic energy of the incident neutron for fission to occur? **[3]**

e) What are the typical kinetic energies of thermal and fast neutrons? Draw a diagram showing how the following cross sections vary with incident neutron energy

- **i.** $^{235}_{92}U(n,f)$
- **ii.** $^{235}_{92}U(n,\gamma)$
- **iii.** $^{238}_{92}U(n,f)$

Use this to explain what the process of moderation is and why it is a critical aspect in the design of a nuclear reactor. **[10]**

f) The deuteron has a mass of 2.014102 u. Calculate its binding energy in MeV. Explain why the fusion reaction $^1H + ^1H \rightarrow ^2He$ does not occur and so direct proton fusion can only proceed through the reaction $^1H + ^1H \rightarrow ^2H + e^+ + \nu_e$. In the solar fusion cycle why is this step known as a bottleneck? The hydrogen burning fusion process can occur through three distinct routes (in the absence of carbon), each involving one of the following partial steps:

- **i.** $^1H + ^1H \rightarrow ^2H + e^+ + \nu_e$
- **ii.** $^7Be + e^- \rightarrow ^7Li + \nu_e$
- **iii.** $^8B \rightarrow ^8Be + e^+ + \nu_e$

Which earth based observations allow researchers to distinguish reaction (ii) from the others? **[7]**

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B3)

a) Explain what is meant by the Extreme Independent Particle Model. [2]

b) Motivate the inclusion of the pairing term in the semi-empirical mass formula, describing its behaviour for even-even, odd-odd and even-odd nuclei. Stable nuclei tend to have Z=N for low A and N>Z for large A. Explain the physical origin of this tendency. [5]

c) Sketch a graph of the behaviour of the neutron absorption cross section as a function of the neutron number. Comment on any notable features. [4]

d) Determine the proton and neutron shell configurations for the following nuclei as well as their expected spin and parity assignments for the ground states.

   i. ${}^{7}_{3}$Li

   ii. ${}^{23}_{11}$Na

   iii. ${}^{33}_{16}$S [6]

e) The nuclear magnetic moment μ for an unpaired proton can be calculated in the Independent Particle Model as

   i. $$\langle\mu\rangle = \left[g_l \frac{j\left(j+\frac{3}{2}\right)}{j+1} - \frac{1}{2}\frac{j}{j+1} g_s \right] \mu_N \quad \text{for } j=l-½$$

   ii. $$\langle\mu\rangle = \left[g_l \left(j-\tfrac{1}{2}\right) + \tfrac{1}{2} g_s \right] \mu_N \quad \text{for } j=l+½$$

Where $g_l = 1$ and $g_s = +5.58$ (in units of μ_N) for a free proton. What is the prediction of this model for the magnetic moment of ${}^{209}_{83}$Bi? Why does this prediction differ from the measured value of $4.1\mu_N$? [6]

f) The average of the neutron absorption cross section for heavy nuclei is approximately 0.1 barn. Would you expect this cross section for ${}^{209}_{83}$Bi to be significantly different from the value of 0.1 barn? [2]

g) The nucleus ${}^{42}_{21}$Sc has an odd number of neutrons and protons. What are the range of ground state spin values that this nucleus could have? [5]

End of Examination Paper

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