Queen Mary
University of London

BSc EXAMINATION

PHY-210 Electric and Magnetic Fields

Time Allowed: 2 hours 15 minutes

Date: 11th May
Time: 10.00

Answer ALL questions in section A. Answer ONLY TWO questions from section B. Section A carries 40 marks, each question in section B carries 30 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question. *The use of numeric calculators is allowed in this examination.*

Data	Permittivity constant	$\varepsilon_0 = 8.85 \times 10^{-12}$	F m^{-1}
	Permeability constant	$\mu_0 = 4\pi \times 10^{-7}$	H m^{-1}
	Electronic charge	$e = 1.60 \times 10^{-19}$	C
	Mass of electron	$m_e = 9.11 \times 10^{-31}$	kg

DO NOT TURN TO THE FIRST PAGE OF THE QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE INVIGILATOR

SECTION A (Answer ALL questions in this section)

A1 (a) Write down a vector equation for the electric field at a distance r from a point charge Q. Define the meaning of all the symbols used. [2]

The x-y plane contains 3 point charges with positions as follows:

A charge of $+Q$ lies at co-ordinates $(a, 0)$.

A charge of $+Q$ lies at co-ordinates $(-a, 0)$.

A charge of $-Q$ lies at co-ordinates $(0,-b)$.

(b) What is the magnitude of the electric field at (0,0)? [5]

A2 (a) Draw the field lines between an electric dipole. [1]

(b) How is the field line density related to the electric field? [1]

(c) State the principle of superposition for the electric field. [2]

(d) What is the value of an electric field inside a perfect conductor? [1]

(e) Define electric flux. [2]

A3 (a) Define electric potential. How is the electric field related to the potential? [3]

(b) The magnitude, E, of the electric field at a perpendicular distance R away from an infinite plane of charge with charge density σ, is $\sigma/(2\varepsilon_0)$. Calculate the electric potential at a perpendicular distance R from the plane, assuming the zero of potential is at the surface of the plane. [3]

A4 (a) What is Ampere's law in mathematical form? [3]

(b) Use it to calculate the magnetic field through the centre of a long solenoid with 1000 turns per meter and a current of 5 amps. [3]

A5 (a) Define the displacement current. [2]

(b) Explain briefly why Maxwell introduced the displacement current and modified Ampere's law. [5]

A6 (a) Write down the expression for the energy density for both an electrical field and a magnetic field. [3]

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(b) For an electromagnetic field given by:

$$\mathbf{E}=\cos(x)\,\mathbf{i} \qquad \mathbf{B}= 1/c\,\sin(x)\,\mathbf{j}\ ,$$

where c is the speed of light. Calculate the total energy density of the electric and magnetic fields. [4]

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SECTION B

B1 (a) State Gauss's law and use it to find the electric field at a perpendicular distance r from from an infinite line of charge with charge per unit length σ. **[8]**

(b) Calculate the work done in moving a charge *q*, at a radial distance of *R* meters from the line to a radial distance of R+3 meters. **[4]**

(c) For a non-conducting cylinder of radius *R*, with constant charge density ρ, what is the electric field inside the cylinder at distance *r* from the cylinder's centre (ie. $R > r$). Plot the magnitude of the electric field for both inside and outside the radius of the cylinder. **[12]**

(d) If an infinite line of charge with charge density σ is placed oriented along the *z*-axis at point (0,*d*) in the x-y plane and another with charge density –σ is placed parallel to the first at point (0,-*d*) what is the electric field at (*x*,0). **[6]**

B2 (a) The Biot-Savart law can be written in the form $d\mathbf{B} = \frac{\mu_0 I d\mathbf{l} \times \hat{\mathbf{r}}}{4\pi r^2}$. Discuss briefly with the aid of diagrams, what this law tells us of the relationship between electric current and magnetic field. **[6]**

(b) A circular loop of wire of radius *a*, centered at the origin and lying in the *y*-*z* plane, carries a current of magnitude *I*. Using the Biot-Savart law, show that the magnitude of the component of the **B** field along the *x*-axis, B_x, is given by

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

[12]

(c) Sketch a graph of B_x versus *x* and indicate on the graph the maximum value of B_x as well as the behaviour of the field in the limit $|x| >> a$. **[4]**

(d) If the single loop of wire in part (b) were replaced by a coil of *N* circular loops, calculate the magnetic moment μ of the coil. Give an expression for the magnetic field component B_x produced by the coil, in terms of μ. **[8]**

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B3 (a) Define capacitance.
Obtain an expression for the capacitance, C for a parallel plate capacitor, with a plate area A and plate separation, d in air. **[8]**

(b) Consider a spherical capacitor consisting of two concentric spheres, the first sphere has radius, a and the second sphere has radius b ($b>a$).
What is the capacitance? **[14]**

(c) What is the energy stored in the capacitor when the voltage difference between the spheres is 4 volts and the radii are given by: b=1cm and a=0.99cm. **[8]**

B4 (a) Define the mutual inductance, M, between two circuits.
Write the relationship between the emf induced in one circuit and the current flowing in the other. **[12]**

(b) A long solenoid has n turns per unit length and carries a current I_1. The magnetic field inside it is uniform with magnitude $B = \mu_0 n I_1$ and direction parallel to the axis of the solenoid. A smaller square coil, with a total of N turns and sides of length a, is placed inside the solenoid with its plane at an angle θ to the direction of the magnetic field.

(i) Show that the mutual inductance, M, between the coil and the solenoid is

$$M = \mu_0 n N a^2 \sin\theta.$$ **[6]**

(ii) The inner coil carries a current I_2. What is the magnitude of its magnetic moment? What are the two possibilities for the direction of the magnetic moment vector? Derive an expression for the magnitude of the torque, τ, which the inner coil experiences. **[6]**

(ii) Calculate the work done in rotating the coil through 180^0, starting at $\theta = 90^0$. Is $\theta = 270^\circ$ a position of stable or unstable equilibrium? Explain your answer. **[6]**

End of Examination Paper **Dr D S Berman**