Queen Mary
University of London

BSc/MSci EXAMINATION

PHY-116 From Newton to Einstein

Time Allowed: 2 hours 15 minutes

Date: 16th May 2007

Time: 10 a.m.

Answer ALL questions in Section A and TWO questions from Section B. Section A carries 40 marks and each question in Section B carries 30 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question.

COMPLETE ALL ROUGH WORKINGS IN THE ANSWER BOOK AND CROSS THROUGH ANY WORK WHICH IS NOT TO BE ASSESSED.

NUMERICAL CALCULATORS ARE PERMITTED IN THIS EXAMINATION.

Data

Acceleration due to gravity	g	10	m s^{-1}
Speed of light	c	3.00×10^8	m s^{-1}

Formulae

Vector dot product	**u.v**	$u_x v_x + u_y v_y + u_z v_z$
Vector cross product	**u×v**	$(u_y v_z - u_z v_y, u_z v_x - u_x v_z, u_x v_y - u_y v_x)$

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR

Section A: Answer ALL Questions

A1. State, giving reasons, whether the following quantities are vectors or scalars:
a) (1, 1, 1)
b) $\left(\frac{\partial}{dx}+\frac{\partial}{dy}+\frac{\partial}{dz}\right)$
c) $1\hat{\mathbf{i}}+0\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ **[3]**

A2. State Newton's Second Law of Motion, and explain what it says about momentum. **[3]**

A3. A ramp is at 30° to the horizontal, and a mass m slides down it with no friction.
a) Indicate on a sketch the force exerted by gravity on the mass, and resolve it into vertical and horizontal components. Give their values.
b) On the same sketch, indicate the components parallel and perpendicular to the ramp of the force exerted by gravity on the mass. Give their values.
c) On a new sketch, indicate the force exerted by the ramp on the mass and give its value.
d) Calculate the acceleration of the mass. **[7]**

A4. A lift cage weighs 500 kg.
a) The cage is lifted through 20 metres vertically. Calculate its gravitational potential energy.
b) The cage is then dropped. Calculate its kinetic energy and velocity when it has fallen 20 metres. **[3]**

A5. A cheerleader has a baton consisting of a light rod one metre long with a ½ kg point mass at each end. Calculate its moment of inertia about
a) an axis perpendicular to the rod and passing through the centre,
b) an axis perpendicular to the rod and passing through one of the masses. **[4]**

A6. State Kepler's Third Law. Derive it for a planet in a circular orbit in an inverse-square law gravitational field. **[5]**

A7. A comet is travelling at 42 km s^{-1} as it crosses Earth's orbit on its way to the Sun. What is its speed later in its orbit when it crosses Earth's orbit again? Explain your reasoning. **[5]**

A8. State Einstein's two Axioms or Principles from which the Special Theory of Relativity may be derived. Which one was known already to Galileo and Newton? **[3]**

A9. A garage is a box 3 m wide (x-direction) by 2 m high (y-direction) and 6 m deep (z-direction). You approach it at a speed $v = ½\sqrt{3}\times c$ (along the z-direction). What dimensions do you observe the garage to have? **[3]**

A10. A clock is stationary in the garage of question A9, and ticks once per second in its rest frame. You approach the garage at the same speed $v=\sqrt{3}/2\times c$ What will you measure the interval between its ticks to be? **[2]**

A11. State the Axiom or Principle, additional to the two for Special Relativity, which leads to the General Theory of Relativity. Give a simple experimental observation that supports it. **[2]**

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Section B: Answer TWO Questions

B1. a) A pendulum consists of a 2 kg bob attached to a high ceiling by a 10 m light rope. It is pulled to one side and released to swing back and forth. As the rope swings through the vertical, the speed of the bob is 14.14 $m.s^{-1}$.
(i) What is the acceleration of the bob, in magnitude and direction, at this instant?
(ii) What is the tension in the rope at this instant?
(iii) How close to the ceiling will the ball be at the end-points of the motion?
[7]

b) A steel hoop weighing 0.5kg rolls without slipping down a ramp inclined at 30° to the horizontal. Calculate its acceleration. [7]

c) Given that $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, the mass of the Earth $M = 5.97 \times 10^{24}$ kg and the radius of the earth is $R = 6380$ km,
(i) Find the escape velocity for a rocket at 500km altitude.
(ii) Find the orbital speed of a satellite in a circular orbit at 500 km altitude
[8]

d) The speed of the Earth in its orbit is approximately 30 km s^{-1}. Given the comet of Question A7, which crosses the Earth's orbit at 42 km s^{-1},
(i) What can you say about the shape of the comet's orbit?
(ii) And what could you say if the comet's speed were 45 km s^{-1}?
[8]

B2. a) With the technology available to the Ancient Greeks, how could they have found
(i) The diameter of the Earth,
(ii) The distance from the Earth to the Moon,
(iii) The distance from the Earth to the Sun? [7]

b) A flywheel with a moment of inertia $I = 10$ kg m^2 is mounted on a light axle. It is initially stationary. A torque of 100 Nm is applied to the axle for 100 s.
(i) Calculate the final angular velocity of the flywheel.
(ii) Calculate the final angular momentum of the flywheel.
(iii) Calculate the final rotational energy of the flywheel.
(iv) If a torque of 10 N m is now applied about an axis perpendicular to the axle of the flywheel, what will the motion be? Explain your answer.
[9]

c) The potential energy of a nitrogen atom in an ammonia (NH_3) molecule varies with position as $U = x^4 - 2x^2$.
(i) Sketch $U(x)$. Give an expression for the force F on the nitrogen atom and sketch $F(x)$.
(ii) At what positions will the nitrogen atom be in stable equilibrium? Explain your answer. [7]

d) A neutron (rest mass 1.675×10^{-27} kg) has a total energy that is twice its rest energy. Find
(i) The kinetic energy of the neutron.
(ii) The magnitude of the momentum of the neutron.
(iii) The speed of the neutron. [7]

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B3. a) An unstable particle is created in the upper atmosphere from a cosmic ray and travels straight down toward the surface of the earth with a speed of $0.999753c$ relative to the Earth. A scientist at rest on the Earth's surface measures that the particle is created at an altitude of 45.0 km.

(i) As measured by the scientist, how much time does it take the particle to travel the 45.0 km to the surface of the Earth?

(ii) Use the length contraction formula to calculate the distance from where the particle is created to the surface of the Earth as measured in the particle's frame.

(iii) In the particle's frame, how much time does it take the particle to travel from where it is created to the surface of the Earth? Calculate this time both by the time dilation formula and also from the distance calculated in part (ii). Do the two results agree? **[7]**

b) Given a pair of events that occur at coordinates (x, y, z, t) and (x', y', z', t') in some inertial frame, explain:

(i) What is meant by the Proper Time Interval between them?

(ii) What is meant by the Proper Length Interval between them?

(iii) Under what conditions can the events be simultaneous?

(iv) Can the pair of events have both a proper time interval and a proper length between them? Give examples to illustrate your answer. **[8]**

c) A polar bear weighing 500 kg runs across the North Pole at 10 ms^{-1} along the line of longitude of Greenwich.

(i) Calculate the magnitude of the Coriolis force tending to deflect the bear from its path.

(ii) State the direction of the Coriolis force of part (i), both *before* the bear reaches the North Pole and *after*.

(iii) For what directions of travel and for what positions on Earth is the Coriolis force on a travelling bear zero? (Note that bears can climb trees.)

[8]

d) Given three vectors **u**, **v** and **w** with magnitudes $u = 1$, $v = 2$ and $w = 2$, and given that the magnitude of the cross product $|\mathbf{u} \times \mathbf{v}| = 1$, find

(i) The angle between **u** and **v**.

(ii) The value of the dot product **u.v**.

(iii) If the value of the triple product $\mathbf{u}.(\mathbf{v} \times \mathbf{w}) = 2$, what are the angles between **u** and **w**, and between **v** and **w**? **[7]**

End of Examination Paper

Prof DJ Dunstan