King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the Authority of the Academic Board.

M.Sci. EXAMINATION

CP/4476 Superconductors, semiconductors and heterostructures

SUMMER 1998

Time allowed: **TWO HOURS**

Candidates must answer any TWO questions. No credit will be given for attempting a further question.

The approximate mark for each part of a question is indicated in square brackets.

TURN OVER WHEN INSTRUCTED

Planck's constant: $h = 6.626\times10^{-34}$ J.s
$\hbar = 1.055\times10^{-34}$ J.s
Boltzmann constant: $k = 1.381\times10^{-23}$ J.K^{-1} ;1 eV/k = 11605 K
Electronic charge: $e = 1.602\times10^{-19}$ C
Mass of an electron: $m_e = 9.110\times10^{-31}$ kg

Answer TWO questions

1) Explain what a vacancy in a lattice is, and how it may arise.

The energy to form a vacancy in a monatomic crystal (displacing the atom to the surface) is 2 eV. What atomic fraction of these defects would be stable at room temperature? Why would you be likely to see higher numbers than this in practice?

[7 marks]

A concentration of 10^{20} m^{-3} vacancies is produced by radiation damage, in each of two identical crystals. When the crystals are heated, the vacancies migrate and are trapped by other static impurities whose concentration in the crystal is much greater than that of the vacancies. Consequently, the rate of loss of the vacancies is proportional to their concentration. After one crystal has been held for 2 hours at 500 K, the concentration of vacancies is measured to be $9.0\text{x}10^{19}$ m^{3}. The second crystal is held at 550 K for 30 minutes and the final concentration is $4.1\text{x}10^{19}$ m^{-3}.

Show that the activation energy of migration of the vacancies is 1.67 eV.

[7 marks]

If a typical phonon frequency in this crystal is 10^{13} s^{-1}, approximately how many jumps do the vacancies have to make before they are trapped by an impurity?

[6 marks]

2) Estimate the binding energy of an exciton in bulk GaAs, using the data at the foot of the page.

[3 marks]

Starting from the expression for the energy of a free particle: $E = \frac{\mathrm{h}^2 k^2}{2m_e}$, derive an expression for the number of states with energy between E and $E+dE$ of an electron constrained so that it can only move freely in two dimensions.

[4 marks]

A quantum well is formed by a 50 Å layer of GaAs sandwiched between thick layers of an AlGaAs alloy which is lattice-matched to GaAs. Given that a particle of mass *m*, in an infinite potential well of width *a*, has energy levels *E* at

$$E = \frac{h^2 n^2}{32ma^2}, \quad n = 1, 2, 3 \ldots,$$

calculate the energies of the photons which are absorbed, and sketch the absorption spectrum of this well, including the excitonic transitions.

[10 marks]

Explain why, when light is incident in a direction perpendicular to the layer, the absorption from the creation of a hole in the $n = 1$ level and an electron in the $n = 2$ level in the well, has a very low intensity.

[3 marks]

[GaAs has an energy gap of 1.4 eV and a dielectric constant, *e*, of 10.9. Its electron, heavy hole and light hole effective masses are 0.066, 0.5 and 0.082 times the free electron mass. AlGaAs has a band gap of 2 eV, and the valence band offset with GaAs is 0.34 eV.

The ionisation energy of a hydrogen atom is: $\frac{m_e e^4}{(2pe_0 h)^2}$ =13.6 eV, where the symbols have their usual meanings.]

3) In a simple form of the BCS theory of superconductivity, the energy gap between the normal and the superconducting level, *e*, is given by:

$$e = 2\mathrm{h}w_D \exp\left(\frac{-2}{VN_F}\right)$$

Explain the meaning of the symbols in this equation. Use it to explain the following features of superconductivity:

i) the isotope effect;

ii) the fact that good metallic conductors are poor superconductors;

iii) the variation of the specific heat of a superconductor with temperature.

[9 marks]

Two identical superconductors are separated by an insulating barrier a few tens of nanometres wide. Explain diagramatically how the current varies with the potential difference across the barrier:

i) near absolute zero;

ii) just below the critical temperature.

[5 marks]

The insulating barrier between the identical superconductors is reduced to a few atomic layers. Use the time-dependent Schrödinger equation to show that a current may flow even when there is no potential difference across the barrier; and an alternating current is produced by a constant potential difference (Josephson tunnelling).

[6 marks]

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