MSci 4261 ELECTROMAGNETISM EXERCISES

Week 9

We have found (see lecture notes 9.3) expressions for the magnetic and electric fields produced by an accelerated charged particle.

1. Show that for the case when the acceleration is parallel to the velocity, the electric field far from the charge is given by

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0}\frac{1}{c}\left[\frac{\mathbf{n}\times[\mathbf{n}\times\dot{\boldsymbol{\beta}}]}{(1-\boldsymbol{\beta}\cdot\mathbf{n})^3 R}\right]_{\text{ret}},$$

2. Show that in this case, the Poynting vector far from the charge is

$$\mathbf{S} = \frac{q^2}{4\pi\epsilon_0}\frac{1}{4\pi c}\left[\frac{\dot{\beta}\sin\theta}{(1-\boldsymbol{\beta}\cdot\mathbf{n})^3 R}\right]_{\text{ret}}^2\mathbf{n},$$

where θ is the angle between $\mathbf{n}$ and the common direction of the velocity and acceleration of the particle.

3. With the expression just obtained, $\mathbf{n}\cdot\mathbf{S}$ is the energy per unit area per unit time detected at the observation point at the time t. This was emitted by the particle at the retarded time $t' = t - R(t')/c$. So in a time interval from $t' = T_1$ to time $t' = T_2$ the energy radiated would be

$$E = \int_{t=T_1+R(T_1)/c}^{t=T_2+R(T_2)/c}\mathbf{n}\cdot\mathbf{S}\,dt = \int_{t'=T_1}^{t'=T_2}\mathbf{n}\cdot\mathbf{S}\frac{dt}{dt'}\,dt'.$$

So the power radiated per unit solid angle is

$$\frac{dP(t')}{d\Omega} = R^2\mathbf{n}\cdot\mathbf{S}\frac{dt}{dt'}.$$

Show that for the case so far considered this gives

$$\frac{dP(t')}{d\Omega} = \frac{q^2}{4\pi\epsilon_0}\frac{1}{4\pi c}\frac{\dot{\beta}^2\sin^2\theta}{(1-\beta\cos\theta)^5}.$$

4. Show that this reduces to the Larmor formula, for $\beta << 1$,

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi\epsilon_0}\frac{1}{4\pi c^3}\dot{u}^2\sin^2\theta.$$

5. Without making the non-relativistic approximation, show that the maximum intensity of radiation is observed at the angle

$$\theta_{\text{max}} = \cos^{-1}\left[\frac{1}{3\beta}(\sqrt{1+15\beta^2}-1)\right] \to \frac{1}{2\gamma}.$$

Sketch a polar plot of the angular distribution for the non-relativistic case (the Larmor case), and for $\gamma \approx 2$.