# MSci 4261 ELECTROMAGNETISM EXERCISES

## Week 6: Answers

**1.** Let $z = e^{iq_1 a}$. Consider the lattice points with $y = z = 0$ and $x = 0, a, 2a, \ldots, (N_1 - 1)a$. Considering the sum $\mathcal{F}(\mathbf{q}) = \sum_{ij} e^{i\mathbf{q}\cdot(\mathbf{x}_i - \mathbf{x}_j)}$ for these points, one gets $(1+z+z^2+...z^{N-1})(1+z^{-1}+z^{-2}+...z^{-N+1})$. This is equal to $\frac{\sin^2(N_1 q_1 a/2)}{\sin^2(q_1 a/2)}$. This is the first factor in the desired formula. The others follow simply by now considering the other lattice points with other values of $y, z$.

**2.** The $k^4$ dependence in the formula means that in the visible spectrum the red is scattered least and the blue the most (and with higher intensity). Thus there is more blue in the light which is received away from the direction of the incident beam. Conversely, there is more red (and weaker intensity) in the transmitted beam. This explains the phenomena mentioned.

**3.** Firstly, the incident wave at the first slab is always $180^0$ out of phase with that at the second slab. It is then scattered coherently, hence the scattered amplitude is zero. When there are density fluctuations, if there are $N + \delta N_1$ molecules in the first layer of air and $N + \delta N_2$ in the second, then the amplitude of scattered radiation produced by each volume is proportional to the number of molecules. Since the radiations differ in phase by $180^0$, the net amplitude is proportional to the difference $(N + \delta N_1) - (N + \delta N_2) = \delta N_1 - \delta N_2$. The intensity is proportional to the square of this. On average, $\delta N_1^2 = \delta N_2^2 = N$, and $\delta N_1 \delta N_2 = 0$, since the fluctuations are uncorrelated and are about zero randomly. Thus the net intensity is proportional to $2N$.