MSci 4261 ELECTROMAGNETISM EXERCISES

Week 6

1. Given

$$\mathcal{F}(\mathbf{q}) = \sum_{ij} e^{i\mathbf{q}\cdot(\mathbf{x}_i - \mathbf{x}_j)},$$

prove the following formula given in the lectures for a lattice of sites:

$$\mathcal{F}(\mathbf{q}) = \left[\frac{\sin^2\left(\frac{N_1 q_1 a}{2}\right)}{\sin^2\left(\frac{q_1 a}{2}\right)}\right] \cdot \left[\frac{\sin^2\left(\frac{N_2 q_2 a}{2}\right)}{\sin^2\left(\frac{q_2 a}{2}\right)}\right] \cdot \left[\frac{\sin^2\left(\frac{N_3 q_3 a}{2}\right)}{\sin^2\left(\frac{q_3 a}{2}\right)}\right].$$

(Hint: build up the formula by first considering a one dimensional lattice with sites at points $y = 0, z = 0$ and $x = 0, a, 2a, \ldots, (N_1 - 1)a$. Then consider lattice points for other y then z values.)

2. Consider the formula for the total scattering cross section per molecule

$$\sigma = \frac{2}{3\pi} \frac{k^4}{\rho_N^2} (n-1)^2$$

representing the power scattered out of the incident sunlight per molecule per unit incident flux. How does the formula explain the following phenomena: the blueness of the sky; the redness of the sunset; the waneness of the winter sun; the ease of sunburning at midday in summer?

3. Consider two thin slabs of air, separated by a distance of half the wavelength of perpendicularily incident light. First, assume that the air is a uniform dielectric, so that adjacent molecules would radiate coherently. Consider scattered radiation emitted at right angles to the incident radiation. What is its amplitude? Now assume density fluctuations $\delta N_1, \delta N_2$ in the layers of air. Using statistical mechanical results for δN^2 and δN, argue that the net intensity of scattered radiation is proportional to $2N$, ie that the density fluctuations make the coherent scattering in air seem incoherent.