MSci 4261 ELECTROMAGNETISM EXERCISES

Week 5: Answers

1. The Lorentz force on the electron is $-e\mathbf{E}(\mathbf{x},t)$, which should be added to the other forces to give the total force to be equated to the mass times acceleration of the electron. So the equation of motion for the electron is

$$m\ddot{\mathbf{x}}(t) = -e\mathbf{E}(\mathbf{x},t) - m\omega_0^2\mathbf{x}(t) - m\beta\dot{\mathbf{x}}(t).$$

[Note that the l.h.s. of this equation would need to be modified if the speed $|\dot{\mathbf{x}}(t)|$ was not very much less than c, the speed of light. It would then read $\dot{\mathbf{p}}(t)$, with $\mathbf{p}(t) = m\gamma\mathbf{v}(t)$, and of course $\mathbf{v}(t) = \dot{\mathbf{x}}(t)$.]

2. We may write $\mathbf{E}(\mathbf{x},t) = \mathbf{E}(\mathbf{x})e^{-i\omega t}$, and then recognise that (neglecting transients) all time-dependent quantities have the same exponential factor. Furthermore, the electron oscillates about the origin, and never moves far from it, so to good approximation we may regard $\mathbf{E}(\mathbf{x})$ as independent of $\mathbf{x}$. So, writing $\mathbf{x}(t) = \mathbf{x}e^{-i\omega t}$, the equation of motion gives

$$-m\omega^2\mathbf{x} = -e\mathbf{E} - m\omega_0^2\mathbf{x} + mi\omega\beta\mathbf{x},$$

from which

$$\mathbf{x} = -\frac{e}{m}\mathbf{E}\frac{1}{\omega_0^2 - \omega^2 - i\omega\beta}.$$

The dipole moment of the electron about its position of equilibrium (the origin) is then $-e$ times this,

$$\mathbf{p} = \frac{e^2}{m}\mathbf{E}\frac{1}{\omega_0^2 - \omega^2 - i\omega\beta}.$$

3. This simply requires integrating the expression

$$\frac{dP}{d\Omega} = \frac{1}{2}\frac{\mu_0}{16\pi^2 c}|\mathbf{p}|^2\omega^4\sin^2\theta$$

over all directions $d\Omega$, and using the result $\int \sin^2\theta d\Omega = \frac{8\pi}{3}$.

4. Expand the expression

$$\left(\frac{dP}{d\Omega}\right) = \frac{\mu_0 c}{8\pi^2}I^2\left[\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta}\right]^2$$

in powers of $1/kd$, using $kd << 1$. This yields (straightforward details omitted)

$$\frac{\mu_0 c}{8\pi^2}I^2\frac{1}{4}\left(\frac{kd}{2}\right)^4\sin^2\theta,$$

and integrating this over all directions yields the result.