MSci 4261 ELECTROMAGNETISM EXERCISES

Week 5

1. Consider an electron of charge $-e$ and mass m acted on by a harmonic force $\mathbf{F} = -m\omega_0^2\mathbf{x}$ and a damping force $-m\beta\dot{\mathbf{x}}$. Suppose that the electron is also acted on by an electric field $\mathbf{E}(\mathbf{x}, t)$. Neglect magnetic effects. Write down the equation of motion of the electron.

2. Now suppose that the electric field varies harmonically in time with frequency ω as $e^{-i\omega t}$, and that the amplitude of oscillation of the electron is small enough that the electric field can be evaluated at the average position of the electron. What is the dipole moment of the electron about its position of equilibrium?

3. Using the results given in the lecture, show that the total time-averaged power radiated in all directions by pure electric dipole radiation is equal to

$$\frac{\mu_0}{12\pi c}|\mathbf{p}^2|\,\omega^4.$$

4. Recall that the time-averaged power radiated from an antenna is given by

$$\left(\frac{dP}{d\Omega}\right) = \frac{\mu_0 c}{8\pi^2} I^2 \left[\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta}\right]^2.$$

Derive from this the following expression for the total time-averaged power radiated for a short antenna (for which $kd << 1$):

$$\frac{\mu_0 c}{3\pi} I^2 \frac{1}{4}\left(\frac{kd}{2}\right)^4.$$