MSci 4261 ELECTROMAGNETISM EXERCISES

Week 4

1. Show that $\delta(cx) = \frac{1}{c}\delta(x)$, for constant c.

2. Show that potentials of the form (work in the Lorewntz gauge)

$$\mathbf{A} = \mathbf{a}\cos[\omega t - \mathbf{k}\cdot\mathbf{x}], \quad \Phi = a^0\cos[\omega t - \mathbf{k}\cdot\mathbf{x}],$$

   with $\mathbf{a}, a^0$ constant, satisfy the equations for the electromagnetic fields in the absence of sources. How are ω and $\mathbf{k}$ related?

3. What is the condition on $\mathbf{A}$ and a^0 which ensures that these potentials are in Lorentz gauge? Show that it is also possible to impose the *radiation gauge* condition $\boldsymbol{\nabla}\cdot\mathbf{A} = 0$. If $\boldsymbol{\epsilon}$ is a unit 3-vector parallel to $\mathbf{a}$, show that we then have $\boldsymbol{\epsilon}\cdot\mathbf{k} = 0$.

4. Given the conditions thus implied by the Lorentz and radiation gauge as above, obtain expressions for the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, and show that they oscillate with angular frequency ω and amplitudes $E = a\omega$ and $B = ak = \frac{a\omega}{c}$ respectively.

5. Why are the electromagnetic waves so described called *waves*, why are they called *plane* waves, and what is the direction of their propagation?