MSci 4261 ELECTROMAGNETISM EXERCISES

Week 3: Answers

1. From the definition, the energy density is given by $\epsilon_0 E_0^2 \cos^2(\omega t - kz)$, whereas the z component of the energy flux is equal to $\sqrt{\frac{\epsilon_0}{\mu_0}} E_0^2 \cos^2(\omega t - kz)$, hence the first result. This is obvious since the energy of the wave moves with its speed. Working out T^{33} gives $\epsilon_0(-E_z E_z - B_z B_z + \frac{1}{2}(E^2 + B^2)) = \epsilon_0 E_0^2 \cos^2(\omega t - kz)$, so that energy flux is c times the momentum flux. This follows immediately if light is a stream of photons, each having momomentum p and energy cp.

2. The relevant boundary condition is

$$\mathbf{E}_p + \mathbf{E}''_p = \mathbf{E}'_p,$$

where the subscript p refers to the component parallel to the interface. This immediately gives the required equation. If this is true for all x then the exponents in the equation must be equal, which implies the two relations given.

3. This follows directly using the convolution theorem in Fourier analysis, noting that the expression $\int_{-\infty}^{\infty} G(\tau)\mathbf{E}(\mathbf{x}, t - \tau)\, d\tau$ is equal to the Fourier transform of the product $G(\omega)\mathbf{E}(\mathbf{x}, \omega)$, with $G(\omega) = \epsilon_r(\omega) - 1$.