MSci 4261 ELECTROMAGNETISM EXERCISES

Week 3

1. Consider the plane waves

$$\mathbf{E} = E_0 \mathbf{i} \cos(\omega t - kz), \quad \mathbf{B} = \sqrt{\mu_0 \epsilon_0} E_0 \mathbf{j} \cos(\omega t - kz),$$

where $\mathbf{i}, \mathbf{j}$ are unit vectors along the x, y axes respectively. Show that for these plane waves the magnitude of the energy flux (which is in the z direction) is c times the energy density. Why is this physically obvious? Show that the energy flux is also c times the momentum flux (Hint: the only non-zero momentum flux is given by the component T^{33}). Why is this also obvious if we think of light being made from photons?

2. Consider incident, refracted and reflected waves at a matter interface, with

$$\mathbf{E}_{inc} = \mathbf{E}_0 e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x})}, \quad \mathbf{B}_{inc} = \frac{c}{\omega}\mathbf{k} \times \mathbf{E}_{inc},$$
$$\mathbf{E}_{refr} = \mathbf{E}'_0 e^{-i(\omega t - \mathbf{k}'\cdot\mathbf{x})}, \quad \mathbf{B}_{refr} = \frac{c}{\omega}\mathbf{k}' \times \mathbf{E}'_{refr},$$
$$\mathbf{E}_{refl} = \mathbf{E}''_0 e^{-i(\omega t - \mathbf{k}''\cdot\mathbf{x})}, \quad \mathbf{B}_{refl} = \frac{c}{\omega}\mathbf{k}'' \times \mathbf{E}''_{refl}.$$

Assume that the matter interface is at $z = 0$, and that the incident wave has electric field parallel to the $z - x$ plane. Let the angles of incidence, refraction and reflection be $\theta, \theta', \theta''$ respectively. Show that the boundary conditions on the fields $\mathbf{E}$ at the interface imply that

$$-E_0 \cos\theta e^{ikx\sin\theta} + E''_0 \cos\theta'' e^{ikx\sin\theta''} = -E'_0 \cos\theta' e^{ik'x\sin\theta'}$$

must be true for all x. Show that this implies that $\theta = \theta''$ (the law of reflection), and $k\sin\theta = k'\sin\theta'$ (Snell's law).

3. Prove the relation

$$\mathbf{D}(\mathbf{x}, t) = \epsilon_0 \mathbf{E}(\mathbf{x}, t) + \epsilon_0 \int_{-\infty}^{\infty} G(\tau)\mathbf{E}(\mathbf{x}, t - \tau)\, d\tau$$

given in the lectures.