MSci 4261 ELECTROMAGNETISM EXERCISES

Week 2: Answers

1. The first part is again a straightforward application of vector calculus using Maxwell's equations. The significance of the term on the right-hand side of the equation

$$\int_S \mathcal{P} \cdot d\mathbf{S} + \frac{\partial}{\partial t} \int_V \mathcal{E} = -\mathbf{J} \cdot \mathbf{E}$$

is that it represents the work done on the system by the field $\mathbf{E}$, and this equation represents conservation of energy, equating this work done with the rate of change of energy of the system.

2. This problem is to check your mathematical facility in understanding and manipulating common expressions, as well as your ability to prove an important result. The solution requires inserting the definitions and manipulating the resulting expressions. The main intermediate result for this derivation is given in the lectures.

3. The Lorentz force law implies that $\dot{\mathbf{p}} = \frac{e}{c}\mathbf{v} \times \mathbf{B}$. Also, the particle energy is constant ($\mathbf{B}$ does no work). Thus the force law implies that

$$\dot{\mathbf{v}} = \mathbf{v} \times \omega,$$

where $\omega = \frac{e}{mc}\mathbf{B}$. The equation above describes circular motion perpendicular to $\mathbf{B}$, with a uniform translation parallel to $\mathbf{B}$. Integrating the equation shows that the velocity is given by the real part of the equation

$$\mathbf{v} = v_p \mathbf{e}_p + a\omega(\mathbf{e}_1 - i\mathbf{e}_2)e^{-i\omega t},$$

where v_p is the velocity component along the field, $\mathbf{e}_p$ is a unit vector parallel to the field and $\mathbf{e}_1, \mathbf{e}_2$ are the other two orthogonal unit vectors. a is a constant. Integrating this equation again gives the displacement, which is readily seen to be a helix with radius a and a constant displacement.