Queen Mary
University of London

M.Sc. EXAMINATION BY COURSE UNIT

ASTM116 Astrophysical Plasmas

25 May 2005 18:15–21:15

This paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each section.

Calculators are NOT permitted in this examination. Numerical answers where required may be determined approximately, to within factors ~ 5*, or left in terms of trigonometric or other transcendental functions.*

You may quote the following results unless the question specifically asks you to derive it. All notation is standard. Vectors are denoted by boldface type, e.g., **A***, while scalars, including the magnitude of a vector, are in italics, e.g.,* $|\mathbf{E}| = E$.

(i) The Lorentz force on a particle of charge q moving in electric and magnetic fields **E** *and* **B** *respectively is given by*

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

(ii) Maxwell's Equations

$$\begin{aligned} \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \\ \nabla \cdot \mathbf{E} &= \frac{\rho_q}{\varepsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned}$$

where $\mu_0 \varepsilon_0 = 1/c^2$.

(iii) The electric and magnetic fields **E** *and* **B** *as measured in a laboratory frame are related to the fields* $\mathbf{E}'$ *and* $\mathbf{B}'$ *measured in a frame moving relative to the laboraty frame at a velocity* **u** *by the transformation laws*

$$\begin{aligned} \mathbf{E}'_{\parallel} &= \mathbf{E}_{\parallel} \\ \mathbf{B}'_{\parallel} &= \mathbf{B}_{\parallel} \\ \mathbf{E}'_{\perp} &= \frac{\mathbf{E}_{\perp} + \mathbf{u} \times \mathbf{B}}{\sqrt{1 - (u^2/c^2)}} \end{aligned}$$

$$\mathbf{B}'_\perp = \frac{\mathbf{B}_\perp - \dfrac{\mathbf{u}\times\mathbf{E}}{c^2}}{\sqrt{1-(u^2/c^2)}}$$

(iv) The MHD equations for a plasma with electrical conductivity σ:

$$\begin{aligned}
\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{V}) &= 0 \\
\rho\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)\mathbf{V} &= -\nabla p + \frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B} \\
\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)(p\rho^{-\gamma}) &= 0 \\
\frac{\partial\mathbf{B}}{\partial t} &= \nabla\times(\mathbf{V}\times\mathbf{B}) + \frac{1}{\mu_0\sigma}\nabla^2\mathbf{B} \\
\mathbf{E} + \mathbf{V}\times\mathbf{B} &= \mathbf{j}/\sigma
\end{aligned}$$

(vi) The following vector identities and relations

$$\begin{aligned}
\nabla\times(\mathbf{a}\times\mathbf{b}) &= \mathbf{a}(\nabla\cdot\mathbf{b}) + (\mathbf{b}\cdot\nabla)\mathbf{a} - \mathbf{b}(\nabla\cdot\mathbf{a}) - (\mathbf{a}\cdot\nabla)\mathbf{b} \\
(\nabla\times\mathbf{B})\times\mathbf{B} &= (\mathbf{B}\cdot\nabla)\mathbf{B} - \nabla\left(\frac{B^2}{2}\right) \\
\nabla\times(\nabla\times\mathbf{B}) &= \nabla(\nabla\cdot\mathbf{B}) - \nabla^2\mathbf{B} \\
\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) &= (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c} \\
(\mathbf{a}\times\mathbf{b})\times\mathbf{c} &= (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{b}\cdot\mathbf{c})\mathbf{a}
\end{aligned}$$

(vii) The following numerical values of physical constants and parameter values:

Name	symbol	value
Electronic Charge	e	1.6×10^{-19} C
Electron volt	eV	1.6×10^{-19} Joules
Electron mass	m_e	9.1×10^{-31} kg
Proton mass	m_p	1.67×10^{-27} kg
Permeability of free space	μ_0	$4\pi\times10^{-7}$ Henry/m
Permittivity of free space	ε_0	8.85×10^{-12} Farad/m
Speed of light in vacuo	c	3×10^{8} m/s
Earth Radius	R_e	6371 km
Astronomical Unit	AU	1.5×10^{11} m
Solar Radius	$R_\odot$	6.96×10^{8}m

SECTION A

Each question carries 7 marks. You should attempt ALL questions.

1. Describe in one or two sentences the definition, physical meaning and relevance of each of the following terms:

   (a) Magnetic moment of a charged particle

   (b) Electron plasma frequency

2. A particle of mass m and charge q moves non-relativistically in a static, uniform magnetic field $\mathbf{B} \equiv B_o\hat{z}$. Show that the motion consists of a constant z-velocity and cyclotron motion around $\mathbf{B}$ with a radius of $r_L = mv_\perp/|q|B_o$. [You may use any approach you wish, e.g., a specific mathematical solution, verification of a trial solution, etc.] A static uniform electric field $\mathbf{E} \equiv E_o\hat{y}$ is added. Explain qualitatively, with the help of a diagram, why the particle motion now has an additional uniform drift. Indicate the direction of the drift, and how it depends on the sign of the particle's charge.

3. Derive the MHD induction equation for $\partial B/\partial t$ for a non-relativistic plasma. Define the Magnetic Reynolds number R_m. Explain how it is related to the MHD induction equation and describe the limiting behaviour of plasmas with small and large values of R_m.

4. Explain what is meant in MHD by "flux freezing." Give two astrophysical examples, including a brief explanation or description (no more than three or four sentences each should be enough) of how flux freezing manifests itself there and what the consequences are.

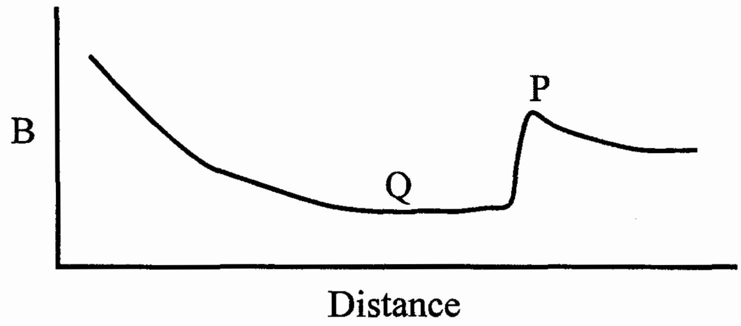
5. The interplanetary magnetic field has a characteristic spiral pattern. Explain how this pattern comes about. Describe how the pattern varies with heliocentric latitude (i.e., from solar equator to poles), and with distance from the Sun.

6. Explain what is meant by "magnetic reconnection." Draw a diagram of the configuration proposed by Petschek to explain fast magnetic reconnection. Include on your diagram indications of the magnetic field and bulk velocity vectors in all regions of space, and the

[This question continues overleaf...]

location of the diffusion region and slow mode shocks. Give two examples of astrophysical situations where magnetic reconnection is important.

7. Define the magnetic moment of a particle in terms of its pitch angle and energy. The figure shows a particular observed profile of the magnetic field amplitude B along a field line in the solar wind. Observations of electrons of a certain energy at point Q show apparent depletions of particles with pitch angles around 90°. On the other hand the electrons observed at point P have a uniform pitch angle distribution. Starting from the observation at P, describe how the depletion of 90° pitch angle particles may be explained by conservation of magnetic moment. (The energy of the particles is such that they may travel freely from P to Q, without appreciably changing their energy.)



SECTION B

Each question carries 17 marks. You may attempt all questions but only marks for the best three questions will be counted.

1. Comets produce a population of neutral particles into the solar wind which are ionized by the UV radiation from the Sun. Consider such a particle, mass m, which has just been ionized to charge q. Use a coordinate system in which the magnetic field $\mathbf{B}$ is in the $\hat{\mathbf{x}}$ direction, and the solar wind velocity $\mathbf{V}_{\rm SW}$ is in the x–y plane and makes an angle α to $\mathbf{B}$. You can assume that the particle is at rest at the origin at time $t = 0$ when it is ionized.

   (a) [*3 marks*] Assuming that ideal MHD applies, what is the electric field $\mathbf{E}$ in component form?

   (b) [*10 marks*] Starting from the Lorentz force, solve the particle's equations of motion to find its velocity and position in component form. [*Hint:* Start from the equations of

 [This question continues overleaf. . .]

motion in component form. Your solution for the y component of particle velocity v_y, should comprise of terms corresponding to gyration and uniform drift.]

(c) [*4 marks*] What is the kinetic energy of the particle averaged over one gyro-period? Explain how the effect of the cometary ions on the solar wind flow may depend on the angle α.

2. Consider the propagation of small amplitude, parallel propagating ($\mathbf{k} \parallel \mathbf{B}_0$) waves in a uniform static plasma obeying the ideal MHD equations. The background plasma has mass density ρ_0, magnetic field $\mathbf{B}_0 = B_0\hat{z}$, pressure p_0, and sound speed given by $c_s^2 = \gamma p_0/\rho_0$.

(a) [*10 marks*] Find the equation for the perturbed quantity $\delta\mathbf{V}$ in terms of the wave properties $\mathbf{k}$ and ω and the background quantities.

(b) [*3 marks*] Show that there exists a longitudinal wave with $\delta\mathbf{V} \parallel \mathbf{k}$, which has the dispersion relation corresponding to a sound wave.

(c) [*4 marks*] Show also that there exists a transverse wave with $\delta\mathbf{V} \perp \mathbf{k}$, and find its dispersion relation. Show that the transverse magnetic field and velocity components are related by

$$\delta\mathbf{B}_\perp = -\frac{kB_0}{\omega}\delta\mathbf{V}_\perp.$$

Give one example where such waves have been observed in a space plasma, and the evidence used to identify the type of wave.

3. Coronal holes are regions on the Sun of open field lines which guide the expansion of the coronal plasma. A model of this expansion within a coronal hole can be built based on the following assumptions:

- The solution is time-steady.
- The cross-sectional area of the coronal hole as a function of radial distance r from the centre of the Sun is given by the function $A(r)$.
- The plasma is isothermal, follows an ideal pressure law $p = 2nk_BT$, and magnetic forces can be neglected.
- The flow is dominated by its radial component, and depends only on r, so that $\mathbf{V} = V(r)\hat{\mathbf{r}}$.

(a) [*2 marks*] Explain why $nmVA = C$, where C is constant.

(b) [*10 marks*] Show that V is governed by:

$$\left(V^2 - \frac{2k_BT}{m}\right)\frac{1}{V}\frac{dV}{dr} = \frac{2k_BT}{m}\frac{1}{A}\frac{dA}{dr} - \frac{GM_\odot}{r^2}.$$

[This question continues overleaf...]

(c) [*5 marks*] Assume that the right hand side of the above equation is negative at the base of the corona. Show that, for any power law dependence of the area (i.e. $A = Dr^{\alpha}$, with D and α being positive constants), the RHS goes to zero at a value $r = r_c$. Give an expression for r_c in the case of radial expansion $\alpha = 2$. What is the significance of this value of r in terms of the flow? Explain what happens if a coronal hole changes so that it expands faster than radially ($\alpha > 2$).

4. An incompressible plasma of uniform density ρ_0 obeys the MHD equations with finite electrical conductivity σ. The magnetic field is given in cartesian coordinates by

$$\mathbf{B} = \begin{cases} (B(z),0,0) & z > 0 \\ (-B(-z),0,0) & z < 0 \end{cases}$$

as found in the classic magnetic field annihilation configuration. Far away from $z = 0$, the field approaches a constant magnitude, B_i (i.e., $B(z \to \pm\infty) = \pm B_i$). The plasma is flowing in towards the $z = 0$ plane, parallel to the z axis with a speed V_i. On either side of $z = 0$ the flow speed can be assumed independent of z.

(a) [*9 marks*] Verify that in steady state the function $B(z)$ is given by

$$B(z) = B_i\left(1 - e^{-\mu_0 \sigma V_i z}\right)$$

(b) [*2 marks*] Hence show that the characteristic thickness ℓ of the region in which the field reverses corresponds to the region where the magnetic Reynolds number, R_m, is of order unity.

(c) [*6 marks*] Describe how reconnection in the Earth's magnetospheric system (for a southward interplanetary magnetic field) can drive flows within the magnetosphere. Use a sketch to show regions where reconnection is expected to occur. Give examples of observations which indicate that reconnection is occurring in the Earth's magnetospheric system.

[End of examination paper]