Queen Mary University of London

M. Sc. Examination by course unit 2010

ASTM108 Cosmology

Duration: 3 hours

Date and time: 20th May 2010, 1815-2115

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You may attempt as many questions as you wish and all questions carry equal marks. Except for the award of a bare pass, only the best 4 questions answered will be counted.

Calculators ARE permitted in this examination. The unauthorized use of material stored in pre-programmable memory constitutes an examination offence. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): J. E. Lidsey

You are reminded of the following information, which you may use without proof:

The following constants may be assumed:

Speed of light, $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$

Gravitational constant, $G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Boltzmann's constant, $k_B = 1.38 \times 10^{-23}\,\mathrm{JK^{-1}}$

Radiation constant, $\alpha = 7.565 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$

Proton mass–energy, $m_p c^2 = 938.3\,\mathrm{MeV}$

Neutron mass–energy, $m_n c^2 = 939.6\,\mathrm{MeV}$

Mega Parsec, $1\,\mathrm{Mpc} = 3.09 \times 10^{22}\,\mathrm{m}$

Hubble time, $H_0^{-1} = 9.8 \times 10^9 h^{-1}\,\mathrm{yr} = 3.09 \times 10^{17} h^{-1}\,\mathrm{s}$

The Conversion Factor, $1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$

The Conversion Factor, $1\,\mathrm{J} = 1\,\mathrm{kg\,m^2\,s^{-2}}$

The following formulae may be assumed:

Friedmann Equation

$$H^2 = \frac{8\pi G}{3}\rho + \frac{8\pi G}{3}\Lambda - \frac{kc^2}{a^2},$$

where $H = \dot{a}/a$ is the Hubble parameter, a is the scale factor of the universe, ρ is the mass density, Λ is the cosmological constant, k is a constant and overdots denote time derivatives.

Conservation Equation

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2}\right) = 0,$$

where p represents the pressure of the matter in the universe.

Acceleration Equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right).$$

Question 1

(a) *[3 marks]* How is the geometry of the universe related to the constant k in the Friedmann equation?

(b) *[6 marks]* Consider a universe that only contains a form of matter with an equation of state $p = \rho c^2$. Determine how the density of this matter would vary with the scale factor of the universe.

(c) *[5 marks]* Assuming $k = \Lambda = 0$ for the universe in part (b), determine how the scale factor would grow with time.

(d) *[6 marks]* With the use of the Friedmann and acceleration equations, describe in brief qualitative terms the complete history of a universe dominated by the type of matter in part (b) if instead $kc^2 = +1$.

(e) *[5 marks]* Suppose instead that a universe contains only a form of matter with an equation of state $p = -\rho c^2$. For $k = 0$, show that such a universe could not have had a definite origin in time.

Question 2

(a) *[3 marks]* Define the critical density, $\rho_c(t)$, and the Ω-parameter, Ω. [Assume $\Lambda = 0$.]

(b) *[6 marks]* Show that the combination of parameters

$$a^2 H^2 (\Omega - 1)$$

is a constant in time. Explain, briefly, why it is important in cosmology to find time-independent quantities involving observable parameters.

(c) *[4 marks]* Observations indicate that the value of Ω today is $\Omega_0 \approx 1.02$. Could Ω become zero at some time in the future? (Explain your reasoning.) Show that a universe with $\Omega = 0$ would expand forever into the future.

(d) *[7 marks]* Assuming that $\Omega_0 \approx 1.02$, calculate the value of Ω at the epoch of decoupling.

(e) *[5 marks]* If inflation occurred in the early universe, describe qualitatively with a sketch how Ω would vary with time before, during, and after inflation. Is a value of $\Omega_0 = 1.02$ consistent with an epoch of inflation?

Question 3

(a) *[4 marks]* Explain what is meant by the isotropy and homogeneity of the universe and state the Cosmological Principle.

(b) *[3 marks]* What is the main observational evidence in favour of the Cosmological Principle?

(c) *[5 marks]* Why is the isotropy of the Cosmic Microwave Background (CMB) temperature viewed as a problem of the hot big bang theory?

(d) *[5 marks]* How does the magnitude of the temperature anisotropies in the CMB provide evidence for the existence of non-baryonic dark matter?

(e) *[3 marks]* What would have been the temperature of the CMB when the universe was one tenth its present size?

(f) *[5 marks]* Derive an expression relating the age of the universe to its temperature that applies before the epoch of matter-radiation equality. [You may assume the universe contained only radiation up to that time.] Use this expression to estimate the universe's age when its temperature was $T \approx 10^8\,\mathrm{K}$.

Question 4

(a) *[4 marks]* What are the conditions in the early universe that determine whether a particle is behaving relativistically or non-relativistically? In each case, state how the mass density of the particle depends on the scale factor.

(b) *[5 marks]* Explain how equilibrium was maintained between the neutrons and protons in the early universe. Further explain why these particles were non-relativistic when they fell out of equilibrium.

(c) *[5 marks]* Describe the process of primordial nucleosynthesis. Why is this regarded as one of the successes of the big bang theory?

(d) *[6 marks]* A theory predicts that stable, very massive particles formed in the early universe just after inflation had come to an end. Suppose these particles came to dominate the density of the universe when the universe was 1 second old. Estimate the density of these particles relative to the critical density at the time of their formation. (You may assume that the universe was dominated by relativistic particles before 1 second.)

(e) *[5 marks]* Suppose the universe is negatively-curved and became dominated by the curvature when its age was $t = 1\,\mathrm{sec}$. By using the Friedmann equation, estimate the temperature of the universe when its age was $t = 3 \times 10^9\,\mathrm{sec}$.

Question 5

(a) *[3 marks]* Define the deceleration parameter, q.

(b) *[6 marks]* Summarize the observational evidence that the present value of q is negative.

(c) *[6 marks]* Write a short note on how observations of the Cosmic Microwave Background radiation over different angular scales can be employed to measure the density of the universe.

(d) *[6 marks]* If, as inferred by observations, the universe is currently undergoing a phase of accelerated expansion, give two explanations as to why it could not always have done so in the past.

(e) *[4 marks]* What problem may arise when determining Hubble's constant from nearby galaxies with low redshifts, $z \ll 1$?

Question 6

(a) *[4 marks]* Explain what is meant by cosmological redshift, z, and state how it is related to the scale factor of the universe.

(b) *[7 marks]* Consider a pressureless, spatially flat universe with a vanishing cosmological constant. Show that the age of such a universe is related to redshift, z, by

$$t(z) = \frac{2}{3}\frac{H_0^{-1}}{(1+z)^{3/2}} \tag{1}$$

(c) *[5 marks]* Given that the temperature of the universe at the epoch of matter-radiation equality was about $30,000\,\mathrm{K}$, use Eq. (1) to estimate the age of the universe at that time.

(d) *[3 marks]* If the age of the oldest globular cluster is 13 billion years, what limit does this imply for the value of the Hubble constant. Why is this limit a problem?

(e) *[6 marks]* Discuss, taking care to explain your reasoning, two modifications that could be made to the cosmological model of part (b) that could enhance the age of the universe for a given observed value of Hubble's constant.

End of Paper