Queen Mary
University of London

MSc EXAMINATION

ASTM-052 Extragalactic Astrophysics

Monday, 23 May 2005 10:00 – 11:30

You may attempt as many questions as you wish and all questions carry equal marks. Except for the award of a bare pass, only the best 2 questions answered will be counted. An ***indicative*** marking scheme is given.

Calculators ARE permitted in this examination, but no programming, graph plotting or algebraic facility may be used. The unauthorised use of material stored in a pre-programmable memory constitutes an examination offence. Please state on your answer book the name and type of machine used.

Physical and Astronomical Constants and Conversion Factors

Speed of light	c	3.00×10^{8}	m s^{-1}
Planck constant	h	6.63×10^{-34}	J s
Gravitational constant	G	6.67×10^{-11}	N m^{2} kg^{-2}
Boltzmann constant	k	1.38×10^{-23}	J K^{-1}
Stefan constant	σ	5.67×10^{-8}	W m^{-2} K^{-4}
Permeability of free space	μ_o	$4\pi \times 10^{-7}$	H m^{-1}
Thomson cross section	σ_T	6.7×10^{-29}	m^{2}
Proton rest mass	m_p	1.673×10^{-27}	kg
Mass of sun	$M_\odot$	1.99×10^{30}	kg
Radius of sun	$R_\odot$	6.96×10^{8}	m
Luminosity of sun	$L_\odot$	3.9×10^{26}	W
Distance of earth from sun	1 AU	1.50×10^{11}	m
Hubble constant	H_o	$100 \times h$	km s^{-1} Mpc^{-1}
Year	1 y	3.16×10^{7}	s
Electron volt	1 eV	1.6×10^{-19}	J
Jansky	1 Jy	10^{-26}	W m^{-2}
Parsec	1 pc	3.085×10^{16}	m

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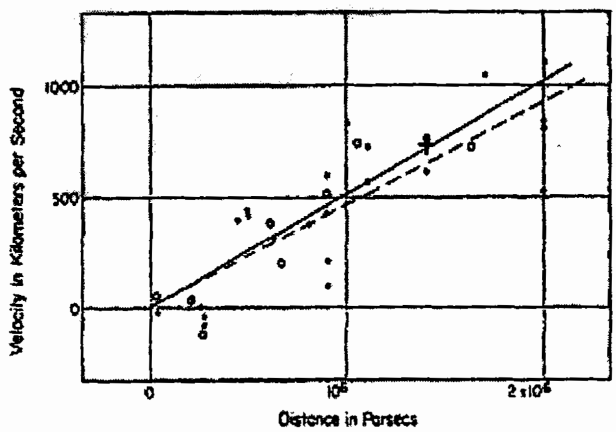
1.



The above figure is a reproduction of Hubble's original plot[1] of the recession velocity of galaxies against their distance. Roughly estimate, from this plot, the value of what came to be called the Hubble constant H_0. Compare your result with current estimates and explain any gross discrepancy. **[5 marks]**

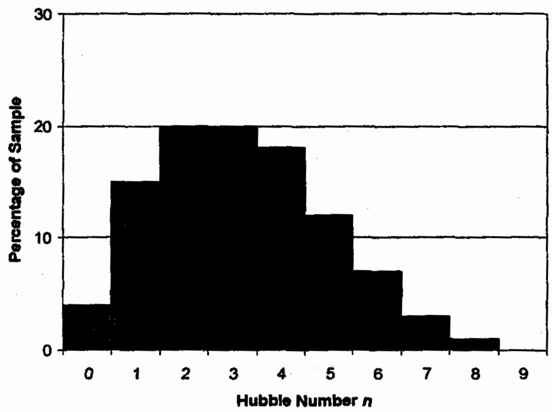
Use a simple argument to show that H_0^{-1} is a measure of the age of the universe and use current values of H_0 to estimate this age. **[7 marks]**

The mass-to-luminosity ratio of spiral galaxies is of the order of $10h$, where

$$H_0 = h \times 100 \text{ km s}^{-1} \text{ Mpc}^{-1}.$$

Explain why h appears in the mass-to-luminosity ratio in this way. **[5 marks]**

Briefly describe Hubble's classification of galaxies. **[5 marks]**



The above figure[2] shows the distribution of elliptical galaxies En as a function of Hubble number n. State briefly why this distribution is inconsistent with all ellipticals' being flat discs, inclined at random angles to the line of sight. **[3 marks]**

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[1] *Proc. Nat. Acad. Sci.,* **15** 165-173 (1929)

[2] Adapted from Lambas, D G, Maddox, S J & Loveday, J., *MNRAS* **258** 404 (1992)

2. State the *virial theorem* for a bound, self-gravitating system and show that, for an elliptical galaxy or a rich cluster of galaxies, it can be approximated by

$$2K + \Omega \approx 0,$$

where K and Ω are the instantaneous values of the kinetic and potential energies of the system.

[5 marks]

Describe in some detail how the masses of elliptical galaxies can be determined. **[10 marks]**

In the solar neighbourhood, the stellar mass-function $\phi(M)$ is given by

$$\phi(M) = \phi_*\left(\frac{M}{M_*}\right)^{-\alpha}; \quad \alpha \sim 2.5,$$

where M is the mass of a star and ϕ_* and M_* are constants. Show that the total mass density M_{total} of stars per unit volume is given by

$$M_{\text{total}} \approx \frac{\phi_* M_*^2}{(\alpha - 2)}\left(\frac{M_{\text{low}}}{M_*}\right)^{2-\alpha},$$

where M_{low} is the lower cut-off of the mass spectrum. **[5 marks]**

The luminosity L of a main sequence star is very roughly proportional to its mass to the power 3.3:

$$L(M) \approx L_*\left(\frac{M}{M_*}\right)^{3.3},$$

where L_* is the luminosity of a star of mass M_*. Show that the total luminosity-density L_{total} of stars in the solar neighbourhood is given by

$$L_{\text{total}} \approx \frac{\phi_* M_* L_*}{(4.3 - \alpha)}\left(\frac{M_{\text{high}}}{M_*}\right)^{4.3-\alpha},$$

where M_{high} is the upper cut-off of the mass spectrum. Discuss the difficulty of using the last two results to obtain the mass-luminosity ratio of stars in spiral galaxies. **[3 marks]**

An estimate of the average mass-luminosity ratio of stars in spiral galaxies is 2, in solar units. Comment on this value in comparison with the observed mass-luminosity ratios of such galaxies. **[2 marks]**

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3. Explain why the motion of stars perpendicular to the plane of a spiral galaxy may be considered to be approximately independent of their motion in the plane, and *vice versa.*

[5 marks]

Discuss why the material making up the spiral arms of a galaxy must be constantly changing.

[7 marks]

The epicyclic angular frequency $\kappa(r)$, at distance r from the centre of a galaxy, is given by

$$\kappa^2(r) = -4B(r)[A(r) - B(r)],$$

where the Oort parameters $A(r)$ and $B(r)$ are given by

$$A(r) = +\frac{1}{2}\left[\frac{\Theta(r)}{r} - \frac{d\Theta(r)}{dr}\right]; \quad B(r) = -\frac{1}{2}\left[\frac{\Theta(r)}{r} + \frac{d\Theta(r)}{dr}\right],$$

$\Theta(r)$ being the circular velocity at r. Obtain an expression for $\kappa(r)$, in terms of the angular frequency $\Omega(r)$ at r, for a galaxy with a flat rotation curve, $\Theta(r)$ = constant. **[5 marks]**

In the case that the random velocity of gas clouds in the disc of a galaxy is zero, the dispersion relation for m-armed spiral density waves of wave-number $k(r)$ at a distance r from the centre of a galaxy can be written as

$$\nu^2(r) > 1,$$

where

$$\nu(r) = \frac{m\,(\Omega_p - \Omega(r))}{\kappa(r)},$$

Ω_p being the angular velocity of the spiral pattern and $\Omega(r)$ being the angular frequency of rotation about the centre. Give a physical explanation of why the waves are disrupted at $\nu(r) = \pm 1$. **[3 marks]**

The dispersion relation can be re-written in the form

$$\Omega(r) - \frac{\kappa(r)}{m} < \Omega_p < \Omega(r) + \frac{\kappa(r)}{m}$$

Define the *Lindblad resonances* and sketch their loci, as a function of pattern speed, for a two-armed galaxy with a flat rotation curve. **[5 marks]**

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4. The soft X-ray luminosity of an active galaxy is observed to double in three hours. Estimate an upper limit to the size of the emitting region. Assuming that this size corresponds to the inner edge of an accretion disc around a black hole, estimate the mass of the hole. [The Schwarzschild radius r_S of a black hole of mass M is given by

$$r_S = \frac{2GM}{c^2},$$

G being the gravitational constant and c being the velocity of light.] **[7 marks]**

Explain qualitatively the origin of the Eddington limit for accretion. **[5 marks]**

The Eddington luminosity $L_{\text{Eddington}}$ of an object of mass M is given by

$$L_{\text{Eddington}} = 4\pi \frac{GMm_p c}{\sigma_T},$$

where m_p is the mass of the proton and σ_T is the Thomson scattering cross section. Deduce that the mass-flow rate $\dot{m}$ through an accretion disc that is radiating at the Eddington limit is given by

$$\dot{m} = \frac{4\pi}{\eta} \frac{GMm_p}{\sigma_T c},$$

where η is the efficiency of converting mass into energy. Estimate this mass-flow for the source discussed above. **[5 marks]**

The luminosity $l(r)$ per unit area of an accretion disc around a black hole of mass M, at distance r from the disc's centre, is given by

$$l(r) = \frac{3}{8\pi} \frac{GM\dot{m}}{r^3} \left[1 - \left(\frac{r_{\text{lso}}}{r} \right)^{1/2} \right],$$

where r_{lso} is the radius of the last stable orbit. Show that the maximum luminosity occurs at a radius of about 1.36 r_{lso}. Give a physical reason for the luminosity's vanishing at r_{lso} itself.

[8 marks]

End of Examination Paper **Peter Clegg**