Answer THREE questions

Mark Allocation

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Units, Masses and Other Values

The convention: $\hbar = c = 1$ will be used throughout this paper.
The values for the following quantities may be assumed in this paper.

Meaning	Value
Masses of u,d,s,c,b,t quarks	1 MeV, 2 MeV, 0.2 GeV, 1.5 GeV, 4.5 GeV, 172 GeV
Masses of e,μ,τ leptons	0.5 MeV, 106 MeV, 1.8 GeV
Mass of all neutrinos	0
Mass of Z boson (M_{Z})	91 GeV
Mass of W boson (M_{W})	80 GeV
Fermi Weak Decay Constant (G_{F})	1.11×10^{-5} GeV^{-2}

CKM Matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.974 & 0.227 & 0.004 \\ 0.227 & 0.973 & 0.042 \\ 0.008 & 0.042 & 0.999 \end{pmatrix}$$

where V_{ij} is the factor for interactions involving quarks i and j.

Dirac Matrices

The Dirac γ matrices satisfy $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ (for μ, ν = 0,1,2,3) and are defined as:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^{i=1,2,3} = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

And the Pauli spin matrices, σ_i, are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which satisfy: $(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{c}) = \vec{a} \cdot \vec{c} + i\vec{\sigma} \cdot (\vec{a} \times \vec{c})$ for 3 component vectors $\vec{a}$, $\vec{c}$.

[Part marks]

1. (a) In the limit that the mass of the W boson, M_W, is the same as the Z boson, M_Z, what would one expect for the relative decay rates for a Higgs boson of mass 400 GeV decaying to W^+W^- and ZZ? **[2]**

   (b) Which two experimental measurements suggest that a Standard Model Higgs boson of mass 400 GeV is strongly disfavoured? **[2]**

   (c) The matrix element squared, $|\mathcal{M}|^2$, for the decay of a Higgs boson to two Z bosons, is:

$$|\mathcal{M}|^2 = \left(\frac{g_W M_Z}{\cos\theta_W}\right)^2 \left(2 + \frac{(p \cdot q)^2}{M_Z^4}\right)$$

   where p and q are the momentum 4-vectors of the Z boson. For a Higgs boson produced at rest show that:

$$|\mathcal{M}|^2 = \left(\frac{g_W M_H^2}{2M_W}\right)^2 \left(1 - \frac{4M_Z^2}{M_H^2} + \frac{12M_Z^4}{M_H^4}\right).$$

   [4]

   (d) Draw the lowest-order, highest-rate Feynman diagram for the decay of a Higgs boson to two gluons. **[3]**

   (e) For $M_H = 400$ GeV, the decay $\mathrm{H} \rightarrow t\bar{t}$ is possible. Considering the decays of the top quark but neglecting subsequent quark decays, estimate what fraction of $\mathrm{H} \rightarrow t\bar{t}$ decays will result in a hadronic final state containing only quarks and anti-quarks. **[6]**

   (f) Experimentally how would one determine whether a hadronic final state was from the decay of a top quark? **[3]**

[Part marks]

2. (a) Draw the lowest-order Feynman diagram for the decay: $K^+ \to l^+ \nu_l$, where $l = e$ or μ. Write down the factors at each interaction vertex. **[6]**

   (b) Consider the decay: $K^+ \to l^+ \nu_l$ in the rest frame of the K^+. Show that the Lorentz gamma factor, $\gamma_l = E_l / m_l$, of the charged lepton, where E_l and m_l are the charged lepton's energy and rest-mass respectively, is given by:

$$\gamma = \frac{m_K^2 + m_l^2}{2 m_l m_K}$$

   where m_K is the mass of the charged kaon. **[6]**

   (c) Hence, explain briefly why the rate of the decay $K^+ \to \mu^+ \nu_\mu$ far exceeds that of $K^+ \to e^+ \nu_e$. **[4]**

   (d) Draw the lowest-order Feynman diagram for the decay $K^+ \to e^+ \nu_e \pi^0$. Neglecting form-factors write down how the decay rate depends on m_K and the CKM elements. **[4]**

   The quark content of the K^+ is $u\bar{s}$.

3. The HERA collider produced head-on collisions between protons of energy E_p and electrons of energy E_e. The square of the 4-momentum transfer in these collisions is defined by $-Q^2$.

   (a) Draw the lowest-order, highest-rate, Feynman diagram describing the production of a single jet (at a large angle with respect to the initial proton direction) in the final state for an $e^- p$ collision with $Q^2 = 100$ GeV^2 at the HERA collider. **[2]**

   (b) Draw three higher order diagrams that could result in two jets being produced in the final state at large angles with respect to the initial proton direction for interactions with $Q^2 = 100$ GeV^2. **[3]**

   (c) Explain why we only observe jets composed of hadrons and not free quarks. **[3]**

   (d) If at HERA collisions occurred along the z axis and the energy and z-component of the momenta of the final state jet(s) are E_H and P_H^z respectively, show that the y variable defined by $\frac{P \cdot q}{P \cdot k}$ can be written as:

$$y = \frac{E_H - P_H^z}{2E_e}$$

   where P and k are the 4-momenta of the colliding proton and electron respectively and q is the 4-momentum transferred in the interaction. You can assume that the masses of the proton and electron can be neglected. **[5]**

   (e) Considering only up and down quarks and their anti-quarks in the proton and the neutron and assuming isospin symmetry, show that:

$$\int_0^1 (F_2^p - F_2^n) \frac{dx}{x} = \frac{1}{3} \int_0^1 (u_V(x) + d_V(x)) + \frac{2}{3} \int_0^1 \left(\overline{u}(x) - \overline{d}(x)\right) dx$$

   where $F_2^N = \Sigma_i e_i^2 x q_i(x)$ and i runs over the quarks and anti-quarks in the nucleon, N, and $q_i(x)$ is the probability that the i^{th} quark (or anti-quark) carries a fraction, x, of the nucleon's momentum. The subscript V denotes valence-quark distributions in the proton.

   Experimentally this quantity is measured to be ≈ 0.24. What does this tell us about the sea-quark structure of the proton? **[7]**

[Part marks]

4. (a) Consider the solution to the Dirac equation, u, defined as:

$$u = \sqrt{|E| + m} \begin{pmatrix} \chi \\ \left(\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\right) \chi \end{pmatrix} \quad \text{where } \chi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and show that $\overline{u}u = 2m$. **[6]**

(b) For a particle, the right-handed state is defined by the projection: $u_R = \frac{1}{2}(1 + \gamma^5)u$ and the left-handed by $u_L = \frac{1}{2}(1 - \gamma^5)u$. Show that $\overline{u}_R = \overline{u}\frac{1}{2}(1 - \gamma^5)$. **[4]**

(c) Hence show that $\overline{u}_R\gamma^\mu u_L = 0$ and interpret the physical significance of this result for EM interactions. **[4]**

(d) Draw the highest-rate, lowest-order Feynman diagram for $e^+e^- \rightarrow \mu^+\mu^-$ at $\sqrt{s} = 91$ GeV and write down the vertex factors. **[4]**

(e) If θ is the angle of the μ^- with respect to the incoming e^- in the e^+e^- centre of mass frame, how would one expect the cross section for the above process to depend on $\cos\theta$ at $\sqrt{s} = 91$ GeV? **[2]**

[Part marks]

5. (a) In QED what is meant by a local gauge transformation? **[2]**

(b) What conditions are required to be satisfied to ensure the QED Lagrangian for a free fermion is invariant under a local gauge transformation? **[2]**

(c) Explain what is meant by the procedure of renormalisation and explain what problem(s) it solves. Illustrate your answer with an appropriate Feynman diagram. **[3]**

(d) The two beams in a circular $\mu^+\mu^-$ collider each have an energy, E_μ, of 500 GeV and cross at a small angle of 250 mrad. Calculate the energy in the centre of mass system assuming $m_\mu \ll E_\mu$. **[3]**

(e) For the muon collider in part (d), assuming only photon exchange interactions, estimate the ratio of the number of interactions producing hadrons in the final state compared to those interactions producing $\tau^+\tau^-$. You need not consider the subsequent decay of the τ leptons. **[4]**

(f) If the muon collider has a radius of 1 km, how many revolutions of the collider will either of the muon beams make before decaying? **[3]**

(g) In addition to the problem of the short lifetime what other issues make a 500 GeV $\mu^+\mu^-$ collider more difficult to realise than a circular 500 GeV e^+e^- collider and which issues make it easier to realise. **[3]**

The lifetime of the muon is $2.2\mu s$.

END OF PAPER