Answer THREE questions

Mark Allocation

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Masses and Other Values

The following symbols may be used in this paper. The following values for these quantities may be assumed for this paper.

Meaning	Symbol	Value
Mass of u quark	$m_{\rm u}$	1 MeV
Mass of d quark	$m_{\rm d}$	2 MeV
Mass of s quark	$m_{\rm s}$	0.2 GeV
Mass of c quark	$m_{\rm c}$	1.5 GeV
Mass of b quark	$m_{\rm b}$	4.5 GeV
Mass of t quark	$m_{\rm t}$	172 GeV
Mass of all neutrinos	m_ν	0
Mass of Z boson	$M_{\rm z}$	91 GeV
Mass of W boson	$M_{\rm w}$	80 GeV
Width of Z boson	$\Gamma_{\rm z}$	2.5 GeV
Weinberg Angle	$\theta_{\rm w}$	28.66^0
Speed of Light	c	3×10^8 ms^{-1}
Fermi Weak Decay Constant	$G_{\rm F}$	1.11×10^{-5} GeV^{-2}
EM Coupling	$\alpha = e^2/(4\pi)$	1/137

Dirac Matrices

The Dirac γ matrices satisfy $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ (for μ, ν = 0,1,2,3) are defined as:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^{i=1,2,3} = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

And the Pauli spin matrices, σ_i, are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which satisfy: $(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{c}) = \vec{a} \cdot \vec{c} + i\vec{\sigma} \cdot (\vec{a} \times \vec{c})$ for 3 component vectors $\vec{a}$, $\vec{c}$.

Lorentz Transformation

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

[Part marks]

1. The CKM unitary matrix gives the flavour-dependent relative couplings for the charged-current weak interactions for quarks and has the following elements:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.974 & 0.227 & 0.004 \\ 0.227 & 0.973 & 0.042 \\ 0.008 & 0.042 & 0.999 \end{pmatrix}$$

where V_{ij} is the factor for interactions involving quarks i and j.

(a) Draw Feynman diagrams for the decays: $B^+ \rightarrow \pi^0 e^+ \nu_e$ and $B^+ \rightarrow \overline{D}^0 e^+ \nu_e$. Ignoring phase space and form-factors, estimate the ratio of the partial widths of these two decay modes. **[7]**

(b) Ignoring phase space and form-factors, estimate the ratio of the partial widths for the two decays: $B^+ \rightarrow \overline{D}^0 e^+ \nu_e$ and $B^+ \rightarrow \overline{D}^0 \pi^+$. **[3]**

(c) In low-energy, semi-leptonic, weak-decays of B(D)-mesons, containing a single $b(c)$-quark, such as the decays in part (a), the relevant energy scale is set by the mass of the meson, M_X. Show, using dimensional arguments, that the semi-leptonic decay rate (Γ) of such mesons, is proportional to M_X^5. **[5]**

(d) The branching ratio for the decays, $B^+ \rightarrow \overline{D}^0 e^+ \nu_e$ and $\overline{D}^0 \rightarrow K^+ e^- \overline{\nu}_e$ are 6.5% and 3.6% respectively. Considering these decays and neglecting form-factors and phase space, show that the lifetime ratio of B^+ to $\overline{D}^0$ mesons, $\frac{\tau(B^+)}{\tau(\overline{D}^0)}$, is expected to be approximately 4. **[5]**

The quark content of the B^+ meson is $u\bar{b}$, that of the $\overline{D}^0$ meson is $\overline{c}u$, that of the K^+ is $\overline{s}u$, that of the π^+ is $u\overline{d}$ and the π^0 is $\overline{u}u$ or $\overline{d}d$.

[Part marks]

2. (a) Given $\hat{H}\psi = i\frac{d\psi}{dt}$ and the Dirac equation $(i\gamma^\mu\partial_\mu - m)\,\psi = 0$.

Show that the Dirac Hamiltonian, $\hat{H}_D$, is given by,

$$\hat{H}_D = -i\gamma^0\left(\vec{\gamma}\cdot\vec{\nabla}\right)\psi + \gamma^0 m$$

and hence that:

$$\hat{H}_D = \begin{pmatrix} m & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -m \end{pmatrix}.$$

[5]

(b) $Y = \overline{u}(a)Xu(b)$, where X is a 4×4 matrix, $u(b)$ is a Dirac spinor and $\overline{u}(a)$ is an adjoint Dirac spinor ($\overline{u} = u^\dagger\gamma^0$) for fermions b and a respectively.

Show, by considering the dimensions of the matrices, that Y is a 1×1 matrix and hence that $Y^* = Y^\dagger$. **[5]**

(c) Show, without using explicit matrix representations, that:

$$Y^* = \overline{u}(b)\gamma^0 X^\dagger\gamma^0 u(a).$$

[5]

(d) Show that $\gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu$ for $\mu = 0, 1, 2, 3$. **[5]**

[Part marks]

3. (a) Draw the Feynman diagram for the scattering process: $\nu_\mu e^- \to \nu_e \mu^-$. **[2]**

   (b) In the laboratory frame the e^- is at rest and the ν_μ has an energy, E_ν. Determine an expression for the total energy, $\sqrt{s}$, in the centre-of-mass (CM) frame, in terms of m_e and E_ν, where m_e is the rest mass of the electron. You should assume that $m_\nu = 0$ and $E_\nu \gg m_\mu$. **[4]**

   (c) The velocity, β, of the CM frame with respect to the laboratory frame is defined by $\beta = \sum \vec{p}_{\mathrm{LAB}} / \sum E_{\mathrm{LAB}}$, where $\sum \vec{p}_{\mathrm{LAB}}$ and $\sum E_{\mathrm{LAB}}$ are the total momenta and energy in the laboratory frame respectively.
   Show that the Lorentz boost, γ, of the CM frame with respect to the laboratory frame is approximately $\sqrt{\frac{E_\nu}{2m_e}}$. **[5]**

   (d) By considering a Lorentz transformation between the CM and laboratory frames defined by $\beta = 1$, $\gamma = \sqrt{\frac{E_\nu}{2m_e}}$, show that the maximum angle (in the laboratory frame), θ_{MAX}, that the μ^- can have with respect to the ν_μ direction in the laboratory frame is given by:

$$\tan\theta_{\mathrm{MAX}} = \sqrt{\frac{2m_e}{E_\nu}},$$

   in the limit that the muon mass can be neglected. **[6]**

   (e) Experimentally, how would one determine the energy of a muon with energy ≈ 10 GeV and distinguish it from an electron? **[3]**

[Part marks]

4. (a) The Lagrangian density of QED is:

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi + e\bar{\psi}\gamma_{\mu}A^{\mu}\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$. Explain with reference to Feynman diagrams what the three terms in the expression for $\mathcal{L}_{\mathrm{QED}}$ correspond to physically. **[3]**

(b) Show that $\mathcal{L}_{\mathrm{QED}}$ is not invariant when ψ undergoes a local phase transformation of the form $\psi \rightarrow \psi e^{ie\theta}$ where θ depends on position. **[5]**

(c) Derive a transformation rule for A_{μ} that restores the invariance of $\mathcal{L}_{\mathrm{QED}}$ under such a local phase transformation. **[2]**

(d) Based on the limits from direct searches and the precision measurements of the W boson and top quark masses and assuming the Higgs mechanism is responsible for electroweak symmetry breaking, what is the approximate upper (at 95% confidence) and lower mass limit of the Higgs boson ?

In the absence of a Higgs signal at the LHC, illustrate with a Feynman diagram what other measurement at the LHC can be used to clarify the mechanism of electroweak symmetry breaking. **[4]**

(e) For a Higgs boson of mass 115 GeV, draw the dominant Feynman diagram for Higgs boson production and decay in proton proton collisions at $\sqrt{s} = 14$ TeV at the LHC. **[3]**

(f) Draw a Feynman diagram for a process that will occur at a far greater rate than the Higgs process in part (e) but will result in the same final state particles. Explain briefly why the rate is so much higher. **[3]**

[Part marks]

5. (a) Draw the Feynman diagram for neutron decay, $n \rightarrow p\, e^- \overline{\nu}_e$. Write down the vertex factors in terms of the weak coupling, g_W, and the Dirac γ matrices. **[6]**

(b) The left-handed state of a particle, u_L, is defined by the projection: $u_L = \frac{1}{2}(1 - \gamma^5)u$ and the adjoint projection by $\overline{u}_L = \overline{u}\frac{1}{2}(1 + \gamma^5)$.
Show that $\left[\frac{1}{2}(1 - \gamma^5)\right]^2 = \frac{1}{2}(1 - \gamma^5)$ and hence that $\overline{u}\gamma^\mu \frac{1}{2}(1 - \gamma^5)u = \overline{u}_L \gamma^\mu u_L$ and interpret the significance of this latter result in the context of neutron decay. **[7]**

(c) Draw the dominant Feynman diagram producing $c\overline{c}$ or $b\overline{b}$ in e^+e^- collisions at $\sqrt{s} = 30$ GeV.
At leading order, neglecting phase-space, what is the value of $R_{c/b}$? $R_{c/b}$ is defined as:

$$R_{c/b} = \frac{\sigma(e^+e^- \rightarrow c\overline{c})}{\sigma(e^+e^- \rightarrow b\overline{b})}.$$

[4]

(d) In approximately what fraction of interactions producing a $b\overline{b}$ would one expect the $b\overline{b}$ to have an additional associated hadronic jet?
Experimentally how would one distinguish between this additional hadronic jet and the particles associated with the $b\overline{b}$ system? **[3]**

END OF PAPER