Answer FOUR questions

Mark Allocation

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Masses and Other Values

The following symbols may be used in this paper. The following values for these quantities may be assumed for this paper.

Meaning	Symbol	Value
Mass of u quark	$m_{\rm u}$	1 MeV
Mass of d quark	$m_{\rm d}$	2 MeV
Mass of s quark	$m_{\rm s}$	0.2 GeV
Mass of c quark	$m_{\rm c}$	1.5 GeV
Mass of b quark	$m_{\rm b}$	4.5 GeV
Mass of t quark	$m_{\rm t}$	172 GeV
Mass of all neutrinos	m_ν	0
Mass of Z boson	$M_{\rm z}$	91 GeV
Mass of W boson	$M_{\rm w}$	80 GeV
Width of Z boson	$\Gamma_{\rm z}$	2.5 GeV
Weinberg Angle	$\theta_{\rm w}$	28.66^0
Speed of Light	c	3×10^8 ms^{-1}
Fermi Weak Decay Constant	$G_{\rm F}$	1.11×10^{-5} GeV^{-2}
EM Coupling	$\alpha = e^2/(4\pi)$	1/137

Dirac Matrices

The Dirac γ matrices satisfy $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ (for μ, ν = 0,1,2,3) are defined as:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^{i=1,2,3} = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

And the Pauli spin matrices, σ_i, are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which satisfy: $(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{c}) = \vec{a}\cdot\vec{c} + i\vec{\sigma}\cdot(\vec{a}\times\vec{c})$ for 3 component vectors $\vec{a}$, $\vec{c}$.

Cross Sections & Natural Units

1 barn $= 10^{-28}\,\mathrm{m}^2$
In natural units $1\,\mathrm{m} = 5.068 \times 10^{15}\,\mathrm{GeV}^{-1}$.

[Part marks]

1. (a) Draw the Feynman diagram for the decay: $\pi^+ \rightarrow e^+\nu_e$. **[3]**

   (b) Define helicity and explain why, for a massive particle, it is not a Lorentz invariant quantity. **[2]**

   (c) The $E > 0$ spinor solution, ψ, to the Dirac equation is:

$$\psi = \begin{pmatrix} \chi \\ \left(\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\right)\chi \end{pmatrix} \quad \text{where } \chi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

   Show that a massless fermion, with spinor, ψ, satisfies:

$$\gamma_5\psi = \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix}\psi$$

   where $\hat{p} = \vec{p}/\,|\,\vec{p}\,|$ **[4]**

   (d) What chiral states of particles and anti-particles participate in the weak interaction? Write an expression for the left-handed chiral eigenstate of a particle in terms of the helicity eigenstates and the particle's velocity, β. **[3]**

   (e) Consider the decay: $\pi^+ \rightarrow l^+\nu_l$ in the rest frame of the π^+. Show that the velocity, $\beta_l = p_l/E_l$, of the charged lepton, where p_l and E_l are the charged lepton's momentum and energy respectively, in this frame satisfies:

$$1 - \beta_l = \frac{2m_l^2}{m_\pi^2 + m_l^2}$$

   [6]

   (f) Hence, explain briefly why the rate of the decay $\pi^+ \rightarrow \mu^+\nu_\mu$ far exceeds that of $\pi^+ \rightarrow e^+\nu_e$. **[2]**

2. The Higgs mechanism allows gauge bosons to have mass without violating local gauge invariance.

   (a) The Lagrangian density for a photon with mass m_γ would be:

   $$\mathcal{L}_\gamma = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$

   where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Show explicitly that $\mathcal{L}_\gamma$ in not invariant under a global gauge transformation:

   $$A^\mu \rightarrow A^\mu + \partial^\mu \Lambda$$

   unless $m_\gamma = 0$. **[3]**

   (b) Draw the Feynman diagram that has the highest cross section for the production and subsequent decay of a Higgs boson of mass 180 GeV at the LHC proton-proton collider. You need not consider any hadronisation processes. **[5]**

   (c) Draw a Feynman diagram for a process that will occur at a far greater rate than the above Higgs process but will result in the same final state particles. Explain briefly why the rate is so much higher. **[3]**

   (d) Draw the Feynman diagram with the largest cross section for the production of a Higgs boson of mass 100 GeV from an e^+e^- collision at $\sqrt{s} \sim 200$ GeV. For this process and considering an e^+e^- collider with counter-circling beams, each with energy E colliding head-on, what is the minimum value of E required to produce a Higgs boson of mass 100 GeV? You may ignore the finite decay width of the Z boson. **[5]**

   (e) For e^+e^- collisions at $\sqrt{s} = 300$ GeV, show that the energy, E_H, of a ($m_H =$ 150 GeV) Higgs boson produced via the process of part (d) is given by:

   $$E_H = \frac{s + m_H^2 - m_Z^2}{2\sqrt{s}},$$

   where m_Z is the mass of the Z boson. **[4]**

3. The CKM unitary matrix gives the flavour-dependent relative couplings for the charged-current weak interactions for quarks and has the following elements:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.974 & 0.227 & 0.004 \\ 0.227 & 0.973 & 0.042 \\ 0.008 & 0.042 & 0.999 \end{pmatrix}$$

where V_{ij} is the factor for interactions involving quarks i and j.

(a) Draw Feynman diagrams for the decays: $B_S \to \pi^+ K^-$ and $B_S \to \pi^+ D_S^-$. Ignoring phase space and form-factors, estimate the ratio of partial widths of these two decay modes. **[6]**

(b) Draw a Feynman containing a $\tau^+ + \nu_\tau$ in the final state from a proton-proton collision at the LHC. **[2]**

(c) What fraction of on-shell W^+ decays at the LHC will result in a τ^+ ? Explain how this fraction would change if off-shell W^+ decays where $(P_W^\mu)^2 > 500$ GeV^2 were considered, where P_W^μ is the 4-vector of the W^+. **[7]**

(d) Draw the Feynman diagram for the decay: $\tau^+ \to \pi^+\pi^0\pi^0\overline{\nu_\tau}$. Experimentally how would one identify the π^0 decay products and reconstruct the π^0 mass? **[5]**

The quark content of B_S is $s\bar{b}$, D_S^- is $\overline{c}s$, K^- is $\overline{u}s$.

[Part marks]

4. (a) Discuss briefly what is meant by the procedure of renormalisation. What problem does it overcome ? **[3]**

(b) The QCD coupling constant, α_s, varies with energy, Q, as follows:

$$\alpha_s(Q^2) = \frac{A}{\ln \frac{Q^2}{\Lambda^2}} \quad \text{where} \quad A = \frac{12\pi}{11n_c - 2n_f},$$

where n_c is the number of colours, n_f is the number of quark flavours and Λ is a fixed energy scale where $\Lambda^2 \ll Q^2$.
With reference to appropriate Feynman diagrams, and to n_c and n_f, explain qualitatively the running of the QCD coupling constant. **[5]**

(c) Give two pieces of experimental evidence that can be used to determine n_c. What experimental measurement can be used to determine the number of light neutrino flavours? **[3]**

(d) For a particle, the left handed state is defined by the projection:
$u_L = \frac{1}{2}(1 - \gamma^5)u$ and the right handed by $u_R = \frac{1}{2}(1 + \gamma^5)u$.
Show that $\overline{u}_L = \overline{u}\frac{1}{2}(1 + \gamma^5)$ **[5]**

(e) Hence show that $\overline{u}_L \gamma^\mu u_R = 0$ and interpret the physical significance of this result for EM interactions. **[4]**

[Part marks]

5. The KEK accelerator produces head-on collisions between 8 GeV e^- and 3.5 GeV e^+.

   (a) What is the centre-of-mass energy, $\sqrt{s}$, of these collisions ? **[3]**

   (b) The leading order cross section for the process $e^+e^- \to b\bar{b}$ is given by:

   $$\frac{d\sigma(e^+e^- \to b\bar{b})}{d\Omega} = \frac{1}{3}\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\Omega},$$

   $$\text{where } \frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\Omega} = \frac{e^4\,(1+\cos^2\theta)}{64\pi^2}$$

   Draw the highest cross section, lowest order Feymnan diagram for $e^+e^- \to b\bar{b}$. Justify the above factor of $\frac{1}{3}$ and explain why the cross sections are proportional to e^4. **[5]**

   (c) Neglecting mass effects, explain, with reference to appropriate leading-order Feynman diagrams, why, at leading order, one expects:

   $$\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\Omega} \neq \frac{d\sigma(e^+e^- \to e^+e^-)}{d\Omega}.$$

   Which cross section should be larger? **[3]**

   (d) Estimate, with reference to the propagator, the ratio of the weak neutral current cross section to the electromagnetic one for $e^+e^- \to b\bar{b}$ at the KEK centre-of-mass energy. **[2]**

   (e) Describe, with reference to particle detectors, how one would identify both the $e^+e^- \to b\bar{b}$ and $e^+e^- \to \mu^+\mu^-$ processes. **[7]**

6. (a) Deep inelastic electron-proton ($e^- p$) scattering can be viewed in terms of a massless parton carrying a fraction, κ, of the proton's 4-momentum exchanging a photon of 4-momentum, q, with the electron. By considering the change in the quark's four-momenta in the quark-photon scattering process, show that:

$$\kappa = \frac{Q^2}{2P \cdot q},$$

where $Q^2 = -q^2$ and P is the proton's 4-momentum. **[3]**

(b) Draw the Feynman diagram for deep-inelastic $e^- p \to e^- X$ scattering. For this process, if the initial electron has energy E_e and the scattered, final state electron has energy E'_e, emerging at an angle θ to the original electron direction, show, for $m_e^2 \to 0$, that:

$$Q^2 = 4E_e E'_e \sin^2 \frac{\theta}{2}$$

[7]

(c) The structure function $F_2(x)$ for electron-nucleon scattering is defined as $\frac{1}{x}F_2(x)^{eN} = \sum_i e_i^2 q_i(x)$ where i runs over all the quarks and anti-quarks in the nucleon, N, and $q_i(x)$ is the probability that the i^{th} quark carries a fraction, x, of the nucleon's momentum. Assuming the proton and neutron are part of an isospin doublet, show that:

$$\frac{F_2(x)^{ep}}{F_2(x)^{en}} = \frac{4u_V(x) + d_V(x)}{4d_V(x) + u_V(x)} \text{ as } x \to 1 \qquad \text{and} \qquad \frac{F_2(x)^{ep}}{F_2(x)^{en}} = 1 \text{ as } x \to 0,$$

where u_V and d_V are the valence up and down quark distributions in the proton, respectively and $F_2(x)^{ep}$, $F_2(x)^{en}$ are the $F_2(x)$ values for electron-proton and electron-neutron scattering respectively. **[8]**

(d) The integral, $\int_0^1 (F_2^{ep}(x) + F_2^{en}(x))\, dx$, is experimentally determined to have a value of 0.3. Neglecting strange and heavier quarks, estimate the fraction of the proton's momentum carried by up and down quarks. **[2]**

END OF PAPER