Answer FOUR questions

Mark Allocation

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Masses and Other Values

The following symbols may be used in this paper. The following values for these quantities may be assumed for this paper.

Meaning	Symbol	Value
Mass of u quark	$m_{\rm u}$	1 MeV
Mass of d quark	$m_{\rm d}$	2 MeV
Mass of s quark	$m_{\rm s}$	0.2 GeV
Mass of c quark	$m_{\rm c}$	1.5 GeV
Mass of b quark	$m_{\rm b}$	4.5 GeV
Mass of t quark	$m_{\rm t}$	172 GeV
Mass of all neutrinos	m_ν	0
Mass of Z boson	$M_{\rm z}$	91 GeV
Mass of W boson	$M_{\rm w}$	80 GeV
Width of Z boson	$\Gamma_{\rm z}$	2.5 GeV
Weinberg Angle	$\theta_{\rm w}$	28.66^0
Speed of Light	c	3×10^8 ms^{-1}
Fermi Weak Decay Constant	$G_{\rm F}$	1.11×10^{-5} GeV^{-2}
EM Coupling	$\alpha = e^2/(4\pi)$	1/137

Dirac Matrices

The Dirac γ matrices satisfy $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ (for μ, ν = 0,1,2,3) are defined as:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^{i=1,2,3} = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

And the Pauli spin matrices, σ_i, are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which satisfy: $(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{c}) = \vec{a} \cdot \vec{c} + i\vec{\sigma} \cdot (\vec{a} \times \vec{c})$ for 3 component vectors $\vec{a}$, $\vec{c}$.

Cross Sections & Natural Units

1 barn = 10^{-28} m^2
In natural units 1 m = 5.068×10^{15} GeV^{-1}.

[Part marks]

1. (a) Particle A has a momentum four-vector P_A and interacts with particle B with momentum four-vector P_B to produce particles C and D with momentum four-vectors P_C and P_D respectively. The rest masses of particles A, B, C and D are m_A, m_B, m_C and m_D respectively. The Mandelstam variables s, t and u are defined by:

$$s = (P_A + P_B)^2; \quad t = (P_A - P_C)^2; \quad u = (P_A - P_D)^2.$$

Show that: $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$. **[4]**

(b) Give one experimental observation for each of the three following statements that provides evidence that quarks:

- are spin 1/2 fermions **[1]**
- are fractionally charged **[1]**
- carry colour. **[1]**

(c) Draw the two lowest order Feynman diagrams with the highest cross section for the scattering of a 2 GeV muon neutrino from a proton resulting in a μ^- in the final state. **[4]**

(d) The differential cross sections for scattering neutrinos, $\sigma(\nu p)$, and anti-neutrinos, $\sigma(\overline{\nu} p)$, with a stationary proton target can be approximated by:

$$\frac{d^2\sigma(\nu p)}{dxdy} = \frac{G^2 xME}{\pi}\left[2d(x) + 2(1-y)^2\overline{u}(x)\right]$$

$$\frac{d^2\sigma(\overline{\nu} p)}{dxdy} = \frac{G^2 xME}{\pi}\left[2\overline{d}(x) + 2(1-y)^2 u(x)\right]$$

where M is the proton rest mass, E is the neutrino or anti-neutrino energy and $u(x)dx, \overline{u}(x)dx, d(x)dx, \overline{d}(x)dx$ represent the number of $u, \overline{u}, d$ and $\overline{d}$ quarks in the proton that carry a fractional momentum in the range $x \rightarrow x + dx$.

Assuming isospin symmetry between the proton and neutron and equality of their masses, show that:

$$\frac{d^2\sigma(\nu d_{np})}{dxdy} = \frac{G^2 xME}{\pi}\left[u(x) + d(x) + (1-y)^2\left(\overline{u}(x) + \overline{d}(x)\right)\right]$$

where d_{np} is a deuteron containing one proton and one neutron and then obtain an expression for $\frac{d^2\sigma(\overline{\nu} d_{np})}{dxdy}$. **[4]**

(e) Integrate over y and x and show that:

$$\sigma(\nu d_{np}) - \sigma(\overline{\nu} d_{np}) = \frac{2G^2 ME}{\pi}$$

[5]

[Part marks]

2. Assume that the free particle ($E > 0$) Dirac spinor solution is:

$$\psi_u^{a,b} = \sqrt{|E| + m} \begin{pmatrix} \chi^{a,b} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{a,b} \end{pmatrix} e^{-ip_\mu x^\mu} \qquad \chi^a = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi^b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and the free particle ($E < 0$) Dirac spinor solution is:

$$\psi_v^{a,b} = \sqrt{|E| + m} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E-m}\phi^{a,b} \\ \phi^{a,b} \end{pmatrix} e^{-ip_\mu x^\mu} \qquad \phi^a = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \phi^b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(a) The charge conjugate spinor of ψ, ψ_C, is defined by

$$\psi_C = i\gamma^2\psi^* \quad .$$

By using the explicit form of the Pauli matrices, determine the charge conjugate spinor of ψ_u^a and show how it is related to ψ_v^a. Explain the significance of this result. **[8]**

(b) Show that if $m \ll E$

$$\gamma^5\psi_u^a \sim \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix} \psi_u^a$$

where $\hat{p} = \vec{p}/|\vec{p}|$ **[4]**

(c) If the projection operators, P_R and P_L, are defined by:

$$P_R = \tfrac{1}{2}(1 + \gamma^5) \qquad P_L = \tfrac{1}{2}(1 - \gamma^5)$$

Show, by constructing a helicity operator, that in the ultra-relatavistic limit, $P_{R,L}$, project out the positive and negative helicity components of a free Dirac spinor. **[5]**

(d) What experimental observation could be used to show that the neutrino is a Majorana and not a Dirac particle ?
Explain why the observation is not allowed for Dirac particles. **[3]**

[Part marks]

3\. (a) What measurements at the Tevatron $p\bar{p}$ collider can be used to constrain the predicted mass of the Higgs boson? **[2]**

(b) Draw the Feynman diagram that has the highest cross section for the production and subsequent decay of a Higgs boson of mass 160 GeV at the Tevatron collider. You need not consider any hadronisation processes. **[5]**

(c) Draw a Feynman diagram for a process that will occur at a far greater rate than the above Higgs process but will result in the same final state particles. Explain briefly why the rate is so much higher. **[4]**

(d) If the Higgs boson decays to two particles A and B of mass m_A and m_B respectively, show that the invariant mass, m_{inv}, of the $A + B$ system and hence of the Higgs boson is given by:

$$m_{\text{inv}}^2 = m_A^2 + m_B^2 + 2\left[E_T^A E_T^B \cosh(\eta_A - \eta_B) - \mathbf{p}_T^A \cdot \mathbf{p}_T^B\right]$$

where $E_T \equiv \sqrt{E^2 - p_z^2}$, $\mathbf{p}_T$ is the transverse momentum 2-vector $= (p_x, p_y)$.

You may assume that:
$E_T \cosh\eta = E$, $E_T \sinh\eta = p_z$, $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

[4]

(e) If the angle between $\mathbf{p}_T^A$ and $\mathbf{p}_T^B$ is $\Delta\phi$ and $\Delta\eta = \eta_A - \eta_B$, show, using the appropriate Taylor expansions, that for massless A and B particles with small $\Delta\phi$ and $\Delta\eta$

$$m_{\text{inv}}^2 \approx |\mathbf{p}_T^A||\mathbf{p}_T^B| \left(\Delta\eta^2 + \Delta\phi^2\right)$$

You may assume that:
$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$; $\cosh(x) = \cos(ix)$ **[3]**

(f) If no Higgs boson is found at the LHC; draw a Feynman diagram of the process one could study to try to elucidate the mechanism of electroweak symmetry breaking. **[2]**

[Part marks]

4. (a) Explain with the help of two Feynman diagrams why the value of the Fermi weak decay constant, G_F^β, deduced from the rate of nuclear β^- decays is slightly less than the value, G_F^μ, deduced from the rate of μ^- decay. Approximately what value would you expect for G_F^β/G_F^μ ? **[4]**

   (b) Explain why the introduction of a phase into the CKM matrix can produce CP violation and why observations of CP violation are important. **[3]**

   (c) Write down the formula for the partial width, Γ_{cb}, for the decay $\overline{B^0} \rightarrow \mathrm{D}^+\mu^-\overline{\nu}_\mu$ in terms of the $\overline{B^0}$ lifetime, τ_B, and the branching ratio, BR, for the decay. How is Γ_{cb} related to V_{cb} ? **[2]**

   (d) Draw a Feynman diagram for the decay $\overline{B^0} \rightarrow \mathrm{D}^+\mu^-\overline{\nu}_\mu$, followed by the decay $D^+ \rightarrow \overline{K_s^0}\pi^+$. Explain briefly, with reference to appropriate particle detectors, how one could identify the D^+ meson in this decay sequence. **[5]**

   (e) The LEP accelerator had 2 counter circling beams that collided electrons and positrons head on. Both beams had an energy of 45.5 GeV. Consider the case of Z production and its subsequent decay to a $b\overline{b}$ pair. Assuming the b quarks form B-meson bound states, how far on average (in cm) would one expect expect each of the B-mesons to travel before decay assuming that the lifetime of B mesons is ~ 1.5 ps? How could one measure this decay distance ? **[6]**

The quark content of $\overline{B^0}$ is $b\overline{d}$, D^+ is $c\overline{d}$, $\overline{K_s^0}$ is $s\overline{d}$ and π^+ is $u\overline{d}$

[Part marks]

5. (a) Draw the two lowest order Feynman diagrams for $e^+e^- \to \mu^+\mu^-$.
Write down fermion coupling expressions for the vertex factors in each diagram in terms of the electromagnetic coupling, g, the Dirac gamma matrices, the Weinberg angle, $\theta_{\rm W}$, and the vector, C_{fV}, and the axial-vector, C_{fA} couplings. **[4]**

(b) If θ is the angle of the μ^- with respect to the incoming e^- in the e^+e^- centre of mass frame, how would one expect the cross section for the above process to depend on $\cos\theta$ for a purely vector interaction? Why is this not observed even for e^+e^- centre of mass energies, $\sqrt{s}$, of 30 GeV ? **[3]**

(c) $A_{\rm FB}$ is a measurement of the angular asymmetry in $\cos\theta$ of μ^- from the above interaction. With reference to the Fermi weak decay constant, $\rm G_F$, the EM coupling, α, and the centre of mass energy, $\sqrt{s}$, obtain a simple expression for the approximate value of $A_{\rm FB}$ for $\sqrt{s} \ll M_z$ and determine a value of $A_{\rm FB}$ at $s = 900$ GeV^2. **[5]**

(d) Why do we expect the above formulae for A_{FB} not to be valid for the process $e^+e^- \to e^+e^-$ at $\sqrt{s} = 30$ GeV ? **[2]**

(e) The above formulae for A_{FB} are leading order formulae. Draw a higher order Feynman diagram that would modify the prediction for A_{FB} and could be used to make a prediction for the mass of the top quark. **[2]**

(f) At the Tevatron $\rm p\bar{p}$ collider, W bosons are produced through quark anti-quark annihilation. θ is the angle defined with respect to the proton direction. On average, the u quark carries a greater fraction of the proton's momentum than the d quark. With reference to appropriate Feynman diagrams, explain what one would expect for the distribution of positrons and electrons from W decay as a function of $\cos\theta$. **[4]**

[Part marks]

6. (a) The Lagrangian density for free electrons is:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

Show that $\mathcal{L}$ is invariant under global gauge (phase) transformations but not under local gauge (phase) transformations. **[6]**

(b) Local gauge invariance can be achieved by replacing the derivative ∂_μ with the "covariant derivative"

$$\mathcal{D}_\mu \equiv \partial_\mu - ieA_\mu$$

(e is a constant), as long as A_μ transforms in a certain way. Derive the transformation rules for A_μ and give its physical interpretation. **[6]**

(c) What would be the implications of adding a term of the form $\frac{1}{2}m_A^2 A^\mu A_\mu$ to the QED Lagrangian density? **[3]**

(d) Show that the substitution of the Lagrangian density:

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu$$

into the Euler-Lagrange equation for A_μ gives Maxwell's equations i.e.

$$\partial_\mu F^{\mu\nu} = j^\nu$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. **[5]**

The Euler-Lagrange equation for a variable ϕ is:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

END OF PAPER