

The basic equations of Cosmology

Physical coordinates \mathbf{r} are related to comoving coordinates \mathbf{x} by

$$\mathbf{r} = a(t)\mathbf{x},$$

where the scale factor $a(t)$ is related to the redshift z by

$$a = 1/(1 + z)$$

($a = 1$ at the present epoch).

The expansion of the Universe is determined by the following equations:

(a) The fluid equation describing the evolution of the density ρ of material:

$$\dot{\rho} + 3 \frac{\dot{a}}{a} \left(\rho + \frac{p}{c^2} \right) = 0,$$

where p is the pressure and c is the speed of light.

(b) The Friedmann equation, given the curvature k and the Cosmological Constant Λ :

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}.$$

(c) The acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2} \right) + \frac{\Lambda}{3}.$$

Note: a combination of two of these equations gives the third equation.