3C43 Lasers and Modern Optics

Sample Part A

1

a) Write down the expression of the effective refractive index for propagation of an electromagnetic wave in a uniaxial material with ordinary and extraordinary refractive indices n_o and n_e, respectively, if the propagation direction forms an angle theta with the principal axis z of the material. [3]

b) Calculate the refractive index in the case c above if n_o =1.5, if n_e =1.8, and theta = 45 degrees. [4]

2

a) Write down what is the difference between a positive and a negative uniaxial material. [4]

b) Is the material of question 1 above a positive or negative uniaxial material? [3]

3

Consider an optical system consisting of 2 thin lenses A and B made of the same glass with refractive index n = 1.6 and with radii $R_{A1} = -R_{A2} = 6$ cm and $R_{B1} = 8$ cm, $R_{B2} = -12$ cm.

a) Calculate the focal lengths f_A and f_B of the lenses [3]

b) Calculate the ray transfer matrices M_A and M_B for both lenses. [3]

4

Consider a thick convex lens with radii R_{A1}, R_{A2} and refractive index n.

a) Define, in words, the cardinal points and planes, and the input and output planes. [6]

5

A laser cavity is constituted by a planar Fabry-Perot etalon whose mirrors have reflectivity $R_1 = 0.91$ and $R_2 = R_1/2$ respectively. The mirrors distance is L = 1m. The laser medium is characterised by a loss coefficient $\gamma = 10\ m^{-1}$. The lasing transition is occurring at a wavelength of 8 μm, and it is inhomogeneously broadened with a linewidth $\Delta\nu$ = 50MHz. The Einstein A coefficient of the upper laser level is 150 s^{-1}.

a) Write down an expression for the threshold small signal gain coefficient k_{th}. [3]

b) Calculate the numerical value of k_{th} for the values of R_1, R_2 and γ given above. [2]

6

a) Draw a labelled energy level scheme of the He-Ne laser. Write down a brief comment of the energy level diagram illustrating the working mechanism. [5]

b) Define the Rayleigh range of a laser beam. [4]

Solutions

1

a)

$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}$$

b)

$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2(\pi/4)}{1.5^2} + \frac{\sin^2(\pi/4)}{1.8^2} = \frac{1}{2}\left(\frac{1}{1.5^2} + \frac{1}{1.8^2}\right) = 0.376$$

$$n_e(\pi/4) = 1.6296$$

2

a) An anisotropic uniaxial material is classified as positive if $n_e > n_o$ and negative if $n_e < n_o$.

b) The material of question 1 is a uniaxial positive material because $n_e > n_o$.

3

a) $$\frac{1}{f} = \frac{n_L - n}{n}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_A} = \frac{1.6-1}{1}\left(\frac{1}{6}+\frac{1}{6}\right) = \frac{0.6}{3} = \frac{1}{5}cm^{-1}$$ → $f_A = 5$ cm

$$\frac{1}{f_B} = \frac{1.6-1}{1}\left(\frac{1}{8}+\frac{1}{12}\right) = \frac{5}{24}0.6 = \frac{1}{8}cm^{-1}$$ → $f_B = 8$ cm

b) $$M_A = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{5} & 1 \end{bmatrix}$$

$$M_B = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{8} & 1 \end{bmatrix}$$

4

a) The cardinal points (planes) are:

1. **Focal points** F_1 and F_2: The first focal point F_1 is the point of the optical axis (O.A.) determined by the intersection (of the O.A.) with a ray that emerges from the optical system under consideration parallel to the O.A. Similarly F_2 is the point of the optical axis determined by the intersection (with the O.A.) of a ray that was travelling parallel to the optical axis before entering the optical system. The **focal planes** are the planes containing the focal points and that are perpendicular to the O.A.
2. The **principal planes** (PP_1 and PP_2) are the planes perpendicular to the O.A. also passing through the intersection of the rays defining the first and second focal points, in each case. In other words these are the points (for PP_1 for example) determined by the intersection of a ray passing through F1 and of the corresponding ray emerging from the optical system. In other terms we can think of the principal planes as those "where all the refraction occurs". The **principal points** P1 and P2 are the intersections of the principal planes with the O.A.
3. The **nodal points** are points of the O.A. defined by the condition that any ray directed towards the first nodal point N_1 emerges from the optical system parallel to the incident ray but displaced so that it appears to come from the second nodal point, N_2. The nodal planes are the planes passing through the nodal points and perpendicular to the optical axis.
4. The **input and the output planes** are the planes perpendicular to the O.A. that also contain the vertices V1 and V_2. The latter are defined as the intersections of the first and second refracting surfaces with the O.A.

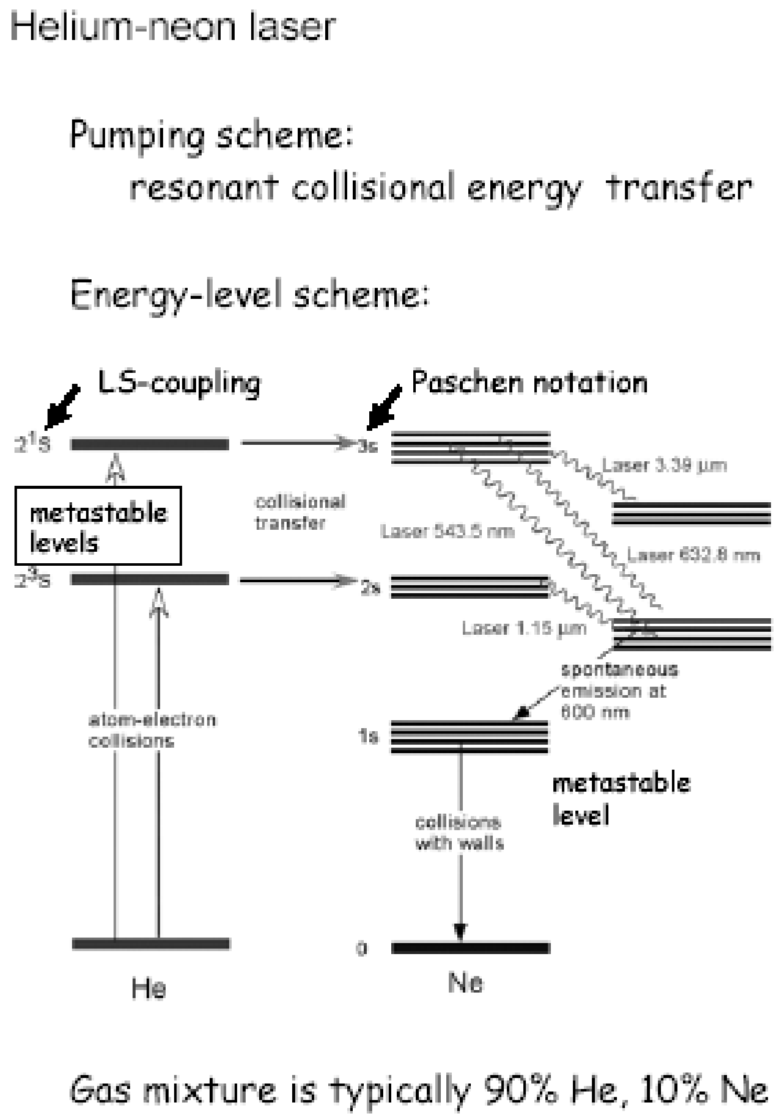
5

a)

$$\kappa_{th} = \gamma - \frac{1}{2L}\ln(R_1 R_2)$$

b)

$$\kappa_{th} = 10 - \frac{1}{2 * 1}\ln(0.91 * 0.455) = 10.44 \quad \text{m}^{-1}$$



In the He-Ne laser the active medium is a mixture of about 1 part of Ne in 9 parts of He. The Ne provides the energy levels for the laser transitions and the He atoms provide an efficient excitation mechanism for the neon ones, although they are not directly involved in the lasing transitions. Excitation is usually provided by a dc discharge created by applying a high voltage (2 to 4 kV) across the gas contained in a narrow diameter glass tube at a pressure of about 10 mbars.

The first step of the pumping process is electron impact excitation of He atoms to one of the 2 metastable states 2^1S and 2^3S. Excited He atoms can then transfer their energy to Ne atoms, with which they may collide, with a probability proportional to $\exp(-\Delta E/kT)$ where ΔE is the energy difference between the excited energy level of the He and one of the excited levels of the Ne (Resonant Energy Transfer), k is Boltzmann constant, and T is the temperature in Kelvin.

The population inversion is created between the 3s and (3p, 2p) levels and also between the 2s and 2p levels. This is a 4 level system. Therefore, electrons in the terminal level should decay to the ground state as rapidly as possible. The transition between the 1s levels and the ground state is facilitated by collisions with the walls of the gas vessel. In order to maintain high efficiency, these lasers have therefore small-bore radii, and this limits their maximum power.