Answer ALL questions from SECTION A and TWO questions from SECTION B

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub–section of a question.

You may assume the following:

$$\int_0^\infty \frac{\partial B}{\partial T}\,\mathrm{d}\nu = \frac{acT^3}{\pi}$$

Gravitation constant $G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Mass of a hydrogen atom $m_\mathrm{H} = 1.67 \times 10^{-27}$ kg

Boltzmann constant $k = 1.38 \times 10^{-23}$ J K^{-1}

Solar radius $R_\odot = 6.96 \times 10^8$ m

Solar luminosity $L_\odot = 3.86 \times 10^{26}$ J s^{-1}

Solar mass $M_\odot = 1.99 \times 10^{30}$ kg

1 yr $= 3.16 \times 10^7$ s

SECTION A

1. Derive expressions for the first three moments of a radiation field of specific intensity I_ν for the plane-parallel approximation. **[7]**

2. List the main assumptions that are usually made in setting up models aimed at describing the interior structure of stars. **[2]**

   Hence, derive the equation of hydrostatic equilibrium. **[4]**

3. Describe what is meant by the assumption of *Local Thermodynamic Equilibrium* (LTE). **[3]**

   Explain why LTE is an adequate approximation deep in a stellar atmosphere, and why LTE breaks down near the top of the atmosphere. **[4]**

4. List the physical processes that give rise to the continuous opacity in stars. **[2]**

   For each source of continuous opacity, describe the conditions needed for it to be significant in a stellar atmosphere and indicate the required effective temperature range. **[5]**

5. Write down the expression for *convective stability* in a star in terms of the adiabatic and radiative temperature gradients. **[2]**

   Thus explain why stars like the Sun have outer convective zones and why massive stars have convective cores. **[5]**

6. The Kelvin-Helmholtz timescale is given by:

$$\tau_{\mathrm{KH}} = \frac{GM^2}{RL}.$$

   Explain the meaning of this timescale in stellar evolution and provide an example of an evolutionary phase that operates on this timescale. **[3]**

   Calculate the thermal timescale (in yrs) for the Sun. **[3]**

SECTION B

7. Explain what is meant by a *diffusive process*. Why can radiative transport in a stellar interior be treated as a diffusive process? **[5]**

The diffusive flux $\vec{\jmath}$ of particles (per unit area and time) between places with differing particle density n is given by

$$\vec{\jmath} = -D\nabla n$$

where the coefficient of diffusion $D = \frac{1}{3}v\ell_{\rm p}$ is determined by the average values of mean velocity v and mean free path $\ell_{\rm p}$ of the particles.

Show that for the case of a stellar interior, where photons are the transporting particles with a radiation energy density $U = aT^4$ and mean free path $\ell_{\rm ph} = 1/(\kappa\rho)$, the equation of radiative transport is given by

$$\frac{\partial T}{\partial r} = -\frac{3}{16\pi ac}\,\frac{\kappa\rho}{r^2T^3}\,L(r)$$

where a is the radiation-density constant, κ is the absorption coefficient, ρ is the density, $L(r)$ is the luminosity, and spherical symmetry has been assumed. **[9]**

Under what conditions in a star does this equation become invalid? **[4]**

By considering the frequency dependence of the flux, and assuming that the radiation energy density $U_\nu = \frac{4\pi}{c}B(\nu,T)$, show that the Rosseland mean absorption coefficient κ_R is given by: **[12]**

$$\frac{1}{\kappa_R} = \frac{\int_0^\infty \frac{1}{\kappa_\nu}\,\frac{\partial B}{\partial T}\,{\rm d}\nu}{\int_0^\infty \frac{\partial B}{\partial T}\,{\rm d}\nu}$$

8. Explain what is meant by a *polytropic process* and give the expression for the polytropic equation of state, explaining the meaning of each term. **[3]**

Write down the polytropic equation of state for (a) a non-relativistic degenerate electron gas, and (b) a relativistic degenerate electron gas. **[4]**

Derive the Lane-Emden equation for a polytrope of index n as given by

$$\frac{1}{z^2}\,\frac{d}{dz}\left(z^2\,\frac{dw}{dz}\right) + w^n = 0$$

where w and z are dimensionless variables defined by $\rho = \rho_c\, w^n$ $(0 \le w \le 1)$; and $z = r/A$ where **[14]**

$$A^2 = \left[\frac{(n+1)K}{4\pi G{\rho_c}^{\frac{n-1}{n}}}\right]$$

Derive the solution for $w(z)$ by integrating the Lane-Emden equation for the case of $n = 0$. **[9]**

9. Describe the main characteristics of *Upper* and *Lower* main sequence stars. **[7]**

Explain the concept of *homology* and describe under what conditions it can be applied to stars. **[4]**

Using the first two equations of stellar structure, derive the first two homology relations, giving the ratios of density and pressure at any point for two stars of mass M_1 and M_2. **[13]**

Hence, show that the homology relation for the temperature ratio is given by **[6]**

$$\frac{T_2}{T_1} = \frac{\mu_2}{\mu_1}\frac{M_2}{M_1}\frac{R_1}{R_2}.$$

10. Write an account of the *pre-main sequence* phase of stellar evolution from the point when hydrostatic equilibrium is established to arrival on the main sequence. Your account should describe the structure and the resulting evolutionary tracks of proto-stars during contraction to the main sequence. **[18]**

Discuss why observations of circumstellar disks, stellar winds and accretion shocks associated with proto-stars have led to revisions of our ideas about the pre-main sequence phase of stellar evolution. **[8]**

The Virial Theorem is given by:

$$2\,U + \Omega = 0$$

where U and Ω are the internal energy and gravitational potential energy and are given by:

$$U = \frac{3}{2}\frac{k\bar{T}M}{\mu m_{\rm H}}; \qquad \Omega = -\frac{GM^2}{R}$$

for a star of mass M and radius R.

Calculate the average Virial temperature $\bar{T}$ of a $1\,{\rm M}_\odot$ proto-star with a radius of $50\,{\rm R}_\odot$ (you may assume that the mean molecular weight $\mu = 0.5$). **[4]**