Answer THREE questions

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub–section of a question.

You may assume the following:

$$\int_0^\infty \left(\frac{\mathrm{d}B_\nu}{\mathrm{d}T}\right)\,\mathrm{d}\nu = \frac{4\sigma T^3}{\pi}$$

Exponential integral:

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n}\mathrm{d}t$$

Mean solar density $\bar{\rho}_\odot = 1400\ \mathrm{kg\,m^{-3}}$

Solar radius $R_\odot = 7.0 \times 10^8\ \mathrm{m}$

Solar luminosity $L_\odot = 3.9 \times 10^{26}\ \mathrm{J\,s^{-1}}$

1. The equation of radiative transfer for a plane-parallel stellar atmosphere is given by (using standard notation):

$$\mu \frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = I_\nu \; - \; S_\nu(\tau_\nu).$$

Derive the formal solution to this equation for I_ν as given by **[6]**

$$I(\tau_\nu, \mu, \nu) = \int_{\tau_\nu}^{\infty} S_\nu(t)\exp[\frac{-(t-\tau_\nu)}{\mu}]\,\frac{dt}{\mu} - \int_0^{\tau_\nu} S_\nu(t)\exp[\frac{-(t-\tau_\nu)}{\mu}]\,\frac{dt}{\mu}.$$

Show that this leads to the following Schwarzschild-Milne relation:

$$J_\nu(\tau_\nu) \quad = \quad \frac{1}{2}\int_0^{\infty} S_\nu(t)\; E_1\,(\;|t-\tau_\nu|\;)\,\mathrm{d}t,$$

where E_1 is the first exponential integral. **[5]**

By assuming that the source function is a linear function of optical depth i.e. $S_\nu(\tau_\nu) = a + b\tau_\nu$ ($b > 0$) show that the emergent flux is given by **[5]**

$$F_\nu(\tau_\nu = 0) = S_\nu(\tau_\nu = 2/3)$$

Write down the three fundamental parameters which describe an LTE stellar model atmosphere. You should include the defining equations where appropriate. **[4]**

2. Explain the meaning of the two terms used in simple convection theory: *mixing length* ℓ_m and *superadiabatic temperature gradient* $(\nabla - \nabla_{ad})$. **[2]**

Show that in the mixing length theory of convection, the convective flux $F_{\rm conv}$ for a star of mass M is given by the equation:

$$F_{\rm conv} = c_p\,\rho\left(\frac{GM}{Tr^2}\right)^{1/2}(\nabla - \nabla_{ad})^{3/2}\,\frac{\ell_m^2}{2}.$$

[8]

Calculate the superadiabatic temperature gradient for the Sun if all the energy transport is by convection (you may assume that $c_p = 3.4\times10^4\,\mathrm{J\,K^{-1}\,kg^{-1}}$, $T_\odot = 10^7\,\mathrm{K}$, $g_\odot = 100\,\mathrm{m\,s^{-2}}$ and $\ell_m = R_\odot/60$). **[5]**

Hence calculate the typical lifetime of a convective cell in days. **[5]**

3. Describe the two main sources of opacity deep inside a stellar interior. **[3]**

Starting with the second moment of the transfer equation for the non-grey and grey cases, derive the expression for the Rosseland mean opacity $\bar{\chi}_R$ as given by: **[10]**

$$\frac{1}{\bar{\chi}_R} = \frac{\pi}{4\sigma T^3} \int_0^\infty \frac{1}{\chi_\nu} \left(\frac{\mathrm{d}B_\nu}{\mathrm{d}T}\right) \mathrm{d}\nu.$$

If the envelope of a star obeys (i) an opacity law of the form $\kappa = \kappa_0 P/T^4$ and (ii) the equation of radiative transport

$$L(r) = -\frac{16\pi ac}{3\kappa\rho} r^2 T^3 \frac{\mathrm{d}T}{\mathrm{d}r},$$

show that the variation of pressure P with temperature T obeys the approximate relation

$$P = \left(\frac{4\pi acGM}{3\kappa_0 L}\right)^{1/2} T^4.$$

You may assume that the star is in hydrostatic equilibrium and that the mass M and luminosity L are constant in the envelope. **[7]**

4. Explain the concept of *homology* and describe under what conditions it can be applied to stars. **[3]**

Derive the two homology relations for the density and pressure ratios at any point for two stars of mass M_1 and M_2, as given by **[10]**

$$\frac{\rho_2}{\rho_1} = \left(\frac{M_2}{M_1}\right)\left(\frac{R_1}{R_2}\right)^3$$

and

$$\frac{P_2}{P_1} = \left(\frac{M_2}{M_1}\right)^2\left(\frac{R_1}{R_2}\right)^4.$$

Calculate the ratio of the pressures of two stars if they have masses of 2 (star 1) and 6 $\mathrm{M}_\odot$ (star 2) and a density ratio $\rho_2/\rho_1 = 0.2$. **[3]**

Explain why we observe two different slopes in the Hertzsprung-Russell diagram for the main sequence on the basis of the appropriate homology relations. **[4]**

5. Describe in general terms how mass loss affects the evolution of stars, quoting specific examples of stellar evolutionary phases where appropriate. **[6]**

Write a detailed account of the predictions of current evolutionary models for a $60\,\mathrm{M}_\odot$ star with mass loss. For each evolutionary phase, your account should include a description of the surface chemical composition and the type of star represented by this composition. **[11]**

Recent theories suggest that the very first stars to form in the Universe may have had masses near $100\,M_\odot$. Considering that they will be composed initially of only hydrogen and helium, suggest three ways in which their evolution will be different from a massive star born today. **[3]**