Memorized equations

Below are a series of equations from the lecture notes that you should memorize. It is important to recognise that this does not define all the material you need to know for your exam: that is defined by the complete set of notes. The other equations within the notes you will be expected to understand, but not memorize. Note that no explanation for the equations is given below. You will need to refer to your lecture notes for that.

Crystal Structure

$$V_c = \left| \vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3) \right|$$

$$\frac{n}{N} \simeq \exp\left(-\frac{E_V}{k_B T} \right)$$

$$\sigma_c = G \frac{a}{2\pi d}$$

Diffraction methods and structure determination

$$2 d \sin\theta = n \lambda$$

$$n(\vec{r}) = \sum_{\vec{G}} n_{\vec{G}} \exp(i \vec{G} \cdot \vec{r})$$

$$n_G = (V_{\text{cell}})^{-1} \int_{V_{\text{cell}}} dV\, n(\vec{r}) \exp(i \vec{G} \cdot \vec{r})$$

$$\vec{G} \cdot \vec{T} = 2\pi (n)$$

$$\vec{b}_i \cdot \vec{a}_j = 2\pi\, \delta_{ij}$$

$$\vec{b}_1 = 2\pi \left(\frac{\vec{a}_2 \times \vec{a}_3}{V_{\text{cell}}} \right); \quad \vec{b}_2 = 2\pi \left(\frac{\vec{a}_3 \times \vec{a}_1}{V_{\text{cell}}} \right); \quad \vec{b}_3 = 2\pi \left(\frac{\vec{a}_1 \times \vec{a}_2}{V_{\text{cell}}} \right)$$

$$\Delta \vec{k} = \vec{G}$$

$$f_G = \int n_{\text{atom}}(\vec{r}) \exp(i \vec{G} \cdot r) \, d^3 \vec{r}$$

$$S_G = \sum_j f_j \exp(i \vec{G} \cdot \vec{r}_j)$$

Cohesion

$$U(R) = 4\epsilon \left[\left(\frac{\sigma}{R} \right)^{12} - \left(\frac{\sigma}{R} \right)^{6} \right] = \frac{A}{R^{12}} - \frac{B}{R^6}$$

Vibrations

$$F_s = M \frac{d^2 u_s}{dt^2} = C(u_{s-1} - u_s) + C(u_{s+1} - u_s) = C(u_{s+1} + u_{s-1} - 2\mathrm{u}_s)$$

$$u_s(t) = (u_s)_0 e^{iKsa} e^{-i\omega t} = (u_s)_0 e^{i(Ksa - \omega t)}$$

$$K_{ZB} = \pm \frac{\pi}{a}$$

$$v_p = \frac{\omega}{K}$$

$$v_g = \frac{d\omega}{dK}$$

$$\epsilon = \left(n + \frac{1}{2} \right) \hbar \omega$$

$$\vec{p}_K = \hbar \vec{K}$$

$$E' = E \pm \hbar\omega(K)$$

$$\vec{k}' = \vec{k} + \vec{G} \pm \vec{K}$$

Thermal Properties

$$U = U_{\text{lattice}} = \sum_{K}\sum_{p} \langle n_{K,p} \rangle \hbar\omega_p(K)$$

$$\langle n \rangle = \frac{1}{\exp(\hbar\omega / k_B T) - 1}$$

$$U = \sum_{\text{p all bands}} \int D_p(\omega) \frac{\hbar\omega}{\exp(\hbar\omega / k_B T) - 1} d\omega$$

$$C_V = \frac{\partial U}{\partial T}$$

$$C_V = 3\mathrm{N} k_B$$

$$K = \frac{m}{N}\frac{\pi}{a} = \frac{m\pi}{L}$$

$$D(K) = \frac{dN}{dK} = 4\pi K^2 \left(\frac{L}{2\pi}\right)^3 = \frac{VK^2}{2\pi^2}$$

$$D(\omega) = \frac{dN}{d\omega} = D(K)\frac{dK}{d\omega} = \frac{VK^2}{2\pi^2}\left(\frac{dK}{d\omega}\right) = \frac{VK^2}{2\pi^2}\frac{1}{v_g}$$

$$\underbrace{\frac{4}{3}\pi K_D^3}_{V_{k\text{-sphere}}} \underbrace{\left(\frac{L}{2\pi}\right)^3}_{1/V_{\text{state}}} = N_{\text{cells}} = N_{DOF}$$

$$C_V = constant \times N k_B \left(\frac{T}{\Theta_D}\right)^3$$

$$\kappa = \frac{1}{3} C_V v_s l$$

Free Electron Gas

$$\epsilon_k = \frac{\hbar^2}{2m} k^2 = \frac{\hbar^2}{2m} \left(k_x^2 + k_y^2 + k_z^2 \right)$$

$$V_{\text{sphere}} = \frac{4}{3} \pi k_F^3$$

$$V_{\text{state}} = \left(\frac{2\pi}{L} \right)^3$$

$$f(\epsilon) = \frac{1}{\exp[(\epsilon - \mu)/k_B T] + 1}$$

$$U = \int_0^{\infty} D(\epsilon) f(\epsilon) \epsilon \, d\epsilon$$

$$C_V = constant \times T$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{F} = \frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt} = \hbar \frac{d\vec{k}}{dt}$$

$$\vec{j} = n q \vec{v} = \frac{n e^2 \tau}{m} \vec{E}$$

$$\sigma = \frac{n e^2 \tau}{m}$$

$$\frac{1}{\tau} = \frac{1}{\tau_i} + \frac{1}{\tau_p}$$

$$j_V = \kappa \frac{dT}{dx}$$

$$\frac{\kappa}{\sigma} = LT$$

Electron Bands

$$\psi(x) = u(x) e^{ikx}$$

$$\epsilon_{\vec{k}} = \langle \vec{k} | H | \vec{k} \rangle = -\alpha - \gamma \sum_m \exp(-\vec{k} \cdot \vec{\rho}_m)$$

Semiconductors

$$v_g = \frac{1}{\hbar} \frac{d\epsilon}{dk}$$

$$\vec{F} = \hbar \frac{d\vec{k}}{dt}$$

$$\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{d^2\epsilon}{dk^2}$$

$$\epsilon_k = E_c + \frac{\hbar^2}{2m_e} k^2$$

$$np = (n_i)^2 = 4 \left(\frac{k_B T}{2\pi\hbar^2} \right)^3 (m_e m_h)^{3/2} \exp\left(-\frac{E_g}{k_B T} \right)$$

Semiconductor Devices

$$np = (n_i)^2$$

$$\sigma = ne\left(\frac{e\tau_e}{m_e}\right) + pe\left(\frac{e\tau_h}{m_h}\right) = ne\mu_e + pe\mu_h$$

$$\mu = \frac{v_g}{E}$$

$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_i}$$

$$-\nabla^2 \phi = \frac{\rho}{\epsilon_r \epsilon_0}$$

$$n_D w_n = n_A w_p$$

$$j = j_s\left(e^{\frac{e\phi}{k_B T}} - 1\right)$$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_p}\right)^2$$

$$\alpha = \frac{I_C}{I_E}$$

$$\beta = \frac{I_C}{I_B}$$

Magnetic Materials

$$E = -\vec{m} \cdot \vec{B}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 \mu_r \vec{H}$$

$$\vec{M} = \chi \vec{H}$$

$$\langle \mu \rangle = \frac{\mu_B e^{\mu_B B / k_B T} - \mu_B e^{-\mu_B B / k_B T}}{e^{\mu_B B / k_B T} + e^{-\mu_B B / k_B T}} = \mu_B \tanh\left(\frac{\mu_B B}{k_B T}\right)$$

$$\chi = \frac{C}{T}$$

$$B_{eff} = \lambda M$$

$$\chi = \frac{C}{T - T_C}$$