Answer SIX questions from SECTION A and THREE from SECTION B.

The numbers in square brackets at the right-hand side of the text indicate the provisional allocation of maximum marks per sub-section of a question.

Permittivity of free space, $\varepsilon_o = 8.85 \times 10^{-12}$ Fm^{-1}

Permeability of free space, $\mu_o = 4\pi \times 10^{-7}$ Hm^{-1}

Speed of light in vacuo, $c = 3.00 \times 10^{8}$ ms^{-1}

Rest mass of proton, $m_p = 1.67 \times 10^{-27}$ kg

Rest mass of electron, $m_e = 9.11 \times 10^{-31}$ kg

Elementary electric charge, $e = 1.60 \times 10^{-19}$ C

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$
$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

SECTION A

1 State Faraday's law of electromagnetic induction. [2]

Show how it leads to the Maxwell equation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$
[5]

2 State the relationship between the magnetic induction **B**, the magnetic field intensity **H** and the magnetization **M**, and give an expression for the relative permeability μ_r in terms of these fields. [2]

Define the magnetization **M**. [2]

Describe with the aid of a sketch an arrangement suitable for measuring the relative permeability of a ferromagnetic material, stating clearly how the permeability is related to the measured quantities. [3]

3 Show that the normal component of **B** and the tangential component of **H** are continuous across a boundary between two different media in which there are no surface conduction currents. [7]

PLEASE TURN OVER

4 Give brief qualitative explanations of diamagnetism and paramagnetism in materials. Classify such materials with respect to their relative permeabilities and indicate how μ_r behaves as a function of temperature in each case. [7]

5 Write down the relationship between the electric field intensity **E**, the displacement field **D** and the electric polarization **P**, and define the electric susceptibility χ_E in terms of these fields. [2]

State the Gauss law for electricity (Maxwell equation 1) in its integral form, defining all symbols. [2]

Hence obtain the differential form for this law. [3]

6 Show that for a plane wave of the form $\mathbf{E} = \mathbf{E}_0\, e^{j(\omega t - \mathbf{k}\cdot\mathbf{r})}$,

$$\nabla \cdot \mathbf{E} = -j\mathbf{k} \cdot \mathbf{E}.$$ [4]

Using this, and the analogous relation

$$\nabla \times \mathbf{E} = -j\mathbf{k} \times \mathbf{E},$$

or otherwise, show that the **E**, **H** and propagation directions of an unbounded plane wave in free space are mutually perpendicular. [3]

7 Draw a ray diagram showing a linearly polarized monochromatic wave striking a flat boundary between two dielectric materials of differing refractive indices, where its **E** vector is normal to the plane of incidence. State which of the two refractive indices is the greater and using the diagram define the angles of incidence, reflection and refraction. [3]

Prove the law of reflection at such a boundary. [4]

8 Outline briefly how a stepped-index optical fibre works. Describe one application where the use of such fibres is appropriate. [3]

Show that propagation is possible along a straight stepped-index fibre provided only that θ, the angle of incidence of the light to the axis of a square-ended fibre, is given by

$$\sin\theta \quad \leq \quad \left(n_1^2 - n_2^2\right)^{1/2},$$

where n_1 and n_2 are the refractive indices of the core and cladding respectively. [4]

[You may confine your argument to rays contained in a plane through the diameter of the fibre]

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SECTION B

9 Define *magnetomotance* $\mathcal{H}$. [3]

Write down the magnetic equivalent of Ohm's law and define the symbols used. What features of magnetic circuits make this equation less precise than Ohm's law for electrical circuits? [3]

Show that the reluctance of a cylinder of material of length ℓ, cross-sectional area *A* and relative permeability μ_r is given by

$$\Re = \frac{\ell}{A\mu_r\mu_o}.$$

What must the orientation of the field be for this formula to be correct? [3]

Obtain the rules for combining two reluctances in series and in parallel. [3]

A C-shaped electromagnet has a yoke (core) of length 4.5 m and uniform cross-sectional area 0.05 m^2 which is made from iron having a relative permeability of 4500. Tapered pole-pieces reduce the cross-sectional area of the air gap to 0.03 m^2 and the gap has a length of 0.04 m. The magnet is energised by a coil having 4000 turns. What current must be applied such that the field in the air gap is 1.5 T? [5]

Estimate the external B–field close to the surface of the yoke. [3]

10 State Ampère's law for steady currents in its differential form. [2]

Write down the continuity equation for electric charge. [2]

Demonstrate that under certain conditions these two equations are inconsistent, stating clearly in words what those conditions are. [4]

Define *displacement current density* and show how the introduction of this quantity is required in order to resolve the above inconsistency. [5]

Show that for a capacitor connected in a circuit, the total displacement current between the electrodes of the capacitor is equal to the conduction current flowing in the circuit. [3]

A circular parallel-plate capacitor has a radius of 10 mm and a gap of 0.1 mm. If the current flowing into the capacitor is 2 A, estimate the B-field between the plates 3 mm from the axis of the capacitor. [4]

PLEASE TURN OVER

11 Write down the Maxwell equations in their differential form which apply to wave propagation within a hollow rectangular wave guide. [4]

Demonstrate that these lead to a wave equation for **E**. [4]

The rectangular wave guide equation may be written

$$\frac{\ell^2}{a^2} + \frac{m^2}{b^2} = \frac{k_o^2 - k_g^2}{\pi^2}.$$

Define the symbols used and show that the product of the phase and group velocities is c^2, where c is the speed of light in free space. [5]

With the aid of a sketch or otherwise, explain what is meant by *TE* and *TM* modes of propagation. Without giving a detailed mathematical proof, indicate in physical terms why the wave guide numbers and m cannot both be zero for either mode. [3]

Show that for a rectangular wave guide of dimensions $8 \times 12\,\mathrm{mm}^2$, radar waves of free wavelength 20 mm may propagate *only* in the TE_{01} mode. Calculate the wavelength within the wave guide. [4]

12 Show that in a cold tenuous plasma containing N free electrons per unit volume, each of charge $-e$ and mass m, the current density **J** produced by an electromagnetic wave of the form $\mathbf{E} = \mathbf{E}_0\, e^{j(\omega t - \mathbf{k}\cdot\mathbf{r})}$ is given by

$$\mathbf{J} = -j\frac{Ne^2}{m\omega}\mathbf{E}.$$ [7]

Hence show that the refractive index of the plasma, n, is given by

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2},$$

where $\omega_p^2 = \dfrac{Ne^2}{\varepsilon_o m}$. [7]

A radio operator finds that at a certain time of day she is able to receive signals from transmitters over the horizon only at frequencies below 300 MHz. Assuming that these signals are reflected or refracted from the ionosphere, what can we deduce about the range of electron density in the upper atmosphere? She also finds that she can receive signals at frequencies of around 1 MHz only from transmitters very far beyond the horizon. Explain, with the aid of a sketch showing ray paths, how this can happen. [6]

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13 For a fixed volume τ surrounded by a surface S, Poynting's theorem states that

$$\oint_S (\mathbf{E}\times\mathbf{H})\cdot\mathbf{dS} \;=\; -\frac{\partial}{\partial t}\int_\tau \tfrac{1}{2}\left(\mathbf{E}\cdot\mathbf{D}+\mathbf{B}\cdot\mathbf{H}\right)\mathrm{d}\tau \;-\int_\tau \mathbf{J}_\mathrm{c}\cdot\mathbf{E}\,\mathrm{d}\tau\,.$$

Explain the physical significance of each of the terms on the right hand side of this equation and hence, from energy conservation, determine the physical significance of the Poynting vector,

$$\mathbf{N} = \mathbf{E}\times\mathbf{H}\,.$$ [5]

The radiation fields produced by an accelerating electric charge are given by

$$\mathbf{E} \;=\; \frac{Q[a]\sin\theta}{4\pi\varepsilon_0\, rc^2}\,\hat{\theta}, \qquad \mathbf{B} \;=\; \frac{Q[a]\sin\theta}{4\pi\varepsilon_0\, rc^3}\,\hat{\phi}.$$

Explain the meaning of all the symbols in the above equations and draw a sketch to indicate the directions of the **E**- and **B**-fields relative to the direction of the acceleration. [3]

Obtain an expression for the total radiated power for a charge with constant acceleration. [6]

A proton moving at one tenth the speed of light enters a piece of material and comes to rest within a distance of 15 mm. Assuming that the deceleration is constant, find the total energy emitted as electromagnetic radiation due to the deceleration, expressing your result as a fraction of the initial kinetic energy of the proton. [6]

END OF PAPER